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Research Article

Influences Of Skewness On The Natural Frequency Of Elastically Supported Skew Laminated Plate Using MQRBF Collocation Approach

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ABSTRACT

This study investigates the influence of skewness on the natural frequency of elastically supported laminated plates via a higher-order shear deformation theory (HSDT) model combined with the multi-quadric radial basis function (MQRBF) collocation method. Five variables displacement field represents the elastically supported skew laminated plate and includes transverse shear deformation effects. The elastic foundation is represented by a two-parameter Pasternak formulation, which addresses normal and shear interactions. The governing equations are developed using Hamilton's principle and discretized using MQRBF. A wide range of parametric studies is conducted to determine the influence of skewing angle, foundation stiffness, span-to-thickness ratio, and orientation on vibration response. The suggested method is confirmed by comparison with known results, proving to be correct and efficient. These findings add to the understanding of the free vibration response of skew laminated plates on elastic foundations, and they can find applications in aerospace engineering, marine engineering, and civil engineering.

1. Introduction

Laminated composite plates are widely applied in aerospace, marine, civil, and mechanical engineering because of their high strength-to-weight ratio, superior stiffness, and flexible material characteristics. In actual applications, laminated composite plates are usually subjected to dynamic loading conditions; their vibration analysis is critical for ensuring structural integrity and performance. Upon being supported on an elastic foundation, their vibration response is determined by parameters like foundation stiffness, material properties, and geometric parameters like skewness. The skew angle has a considerable effect on structural rigidity and the associated natural frequencies of the plate system.

In contrast to traditional rectangular plates, skew plates have complex patterns of deformation because of their non-orthogonal configuration, hence complicating their vibration analysis. The presence of an elastic foundation contributes to increased complexity in the system by changing its stiffness and damping characteristics. Research on skew laminated plates has received growing interest within the research community during the last few years, especially for advanced engineering structures such as aircraft wings, ship decks, and bridges.

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Sufficient studies have been conducted on laminated composite plates, covering topics like bending response and free vibration analysis. [1], thermal bending [2] and thermomechanical responses [3], using refined theories with four unknowns. Generally, these structures are addressed using the Equivalent Single-Layer (ESL) theory. The Classical Laminated Plate Theory (CLPT) [4], being one of the very first ESL models, relies on the negligible transverse shear deformation assumption. Although CLPT works adequately for thin plates, it does not effectively predict the behavior in moderately thick structures, tending to give underestimated deflections and overestimated natural frequencies and buckling loads. To meet this, the First-Order Shear Deformation Theory (FSDT) [5,6] includes effects of transverse shear, albeit based on shear correction factors specific to individual problems. Greater enhancements are observed with HSDTs [7],[8],[9] which involve higher-order terms to increase the accuracy of predictions for displacement and shear stress. HSDT has been identified as a very good tool for predicting the vibration behavior of skew composite plates since it can capture the transverse shear deformation exactly without requiring shear correction factors.

During the last three decades, free vibration responses of skew composite plates, as well as their classical laminated counterparts, have attracted a great deal of attention because of their extensive utilization in the aerospace, automotive, naval, and civil infrastructure industries. Skew plates are even more challenging because their geometric asymmetry influences their distribution of stiffness as well as dynamics. Therefore, numerous researchers have utilized a variety of analytical, numerical, and computational methods to study their vibrational behavior for various boundary conditions and load cases. Kapania and Singhvi [10] performed a thorough analysis of the static and vibration behavior of arbitrarily laminated skew plates with arbitrary edge restraints. Their method applied the Rayleigh–Ritz method based on an energy functional formulation that reflected the anisotropy and complexity of laminated composites. Han and Dickinson [11] advanced this work by creating a Ritz-based solution for symmetrically laminated thin skew plates, thus yielding efficient closed-form approximations of their vibration frequencies. Further extending the numerical modeling functionality, Woo et al.[12] extend the use of the hierarchic degenerated shell element to geometric nonlinear analysis of composite laminated skew plates by the p-version of the finite element method. Liew et al. [13] added the FSDT to a meshless computational framework based on the moving least squares differential quadrature method. The combined method provided an exact prediction of the free vibration properties of moderately thick symmetric laminated plates without relying on a conventional mesh. Park et al.[14] incorporated delamination effects into their vibration analysis via the HSDT model to analyze composite skew plates with centrally positioned quadrilateral cut-outs. Their paper brought out the sensitivity of natural frequencies to internal defects. Kumar and Kumar[15] implemented a meshfree approach for the buckling and vibration analysis of a laminated plate in the framework of the HSDT model. In another significant contribution, Lee [16] considered the dynamic stability of skew laminated structures under in-plane pulsating loads. With the help of finite element modeling using HSDT, he captured intricate modes of deformation that occur as a result of geometric skewness and dynamic instability under time-varying loads. Chun et al., [17] presented analytical solutions for skewed thick plates under transverse loading that have previously been unreported in the literature. The thick plate solution is obtained in a framework of an oblique coordinate system. Zeinali et al.[18] investigated buckling optimization of sandwich plates with trapezoidal corrugated cores and cutouts using vibration correlation techniques and artificial neural networks, highlighting the influence of geometry on stability. Hosseini et al., [19] developed an RBF-based meshless collocation method for vibration analysis of functionally graded plates using FSDT. The method showed high accuracy and efficiency without requiring mesh generation.

Hosseini and Rahimi [20] presented experimental and numerical analysis of hyperelastic plates using the Mooney–Rivlin model with a meshless collocation approach, demonstrating good agreement between experimental and computational results. Wang et al.[21] introduced a new formulation of the differential quadrature method specifically for skew plate geometries. The improved efficiency and accuracy of this method made it an attractive option for high-fidelity vibration analysis. Rajabi and Mohammadimehr [22] studied the bending analysis of a micro sandwich skew plate with an isotropic core and piezoelectric composite face sheets reinforced by carbon nanotubes on elastic foundations. Zeinali et al.[23] proposed a vibration correlation technique combined with machine learning to estimate the buckling load of sandwich plates with circular cores, achieving accurate nondestructive predictions. Kumar et al. [24] investigated laminated skew hyper-shells, a more complicated structural geometry, through a finite element approach based on HSDT. Their findings showed the decisive influence of curvature and shell effects on mode shapes and natural frequencies. Finally, Szekrényes[25] and Sahoo et al. [26]studied vibration properties of composite delaminated plates, highlighting interlaminar damage and its negative influence on structural stiffness and dynamics. Zeinali et al.[27] applied machine learning and genetic algorithms to estimate and optimize buckling load in sandwich beams with honeycomb cores, showing improved prediction and design efficiency.

For complicated geometries like skew plates, traditional numerical techniques like the finite element and finite difference methods frequently require a great deal of meshing and preprocessing, which can be difficult. In this context, meshless methods, particularly the RBF collocation method, offer significant advantages. RBF methods eliminate the need for predefined meshes and allow for high flexibility in handling irregular domains and scattered nodes. The RBF collocation method has demonstrated excellent accuracy and computational efficiency in solving problems related to FGMs and complex geometries. The meshfree technique based on RBF has been utilized for structural analysis by several academics. For instance, the RBF approach for interpolating dispersed data was initially presented by Hardy. [28]. Since then, a lot of work has been done to analyze plates using RBF techniques. Ferreira et al. [29] used a global collocation approach based on MQRBF to study the free vibration of FGM plates. Using the RBF technique, Srivastava et al. [30] investigated the flexural analysis of an elastically supported BDFGM skew plate. Fouaidi et al. [31] used the multiquadric RBF and asymptotic numerical methods for nonlinear and linear static analysis of single-walled carbon nanotubes, with the extension of the applicability of the RBF-based formulation to nanoscale structures. Singh and Shukla [32] conducted nonlinear flexural analysis of laminated composite plates using an RBF-based meshless approach and established its efficacy by comparing the results with those obtained from analytical and FEM-based solutions. Kumar et al. [33] performed a geometrically nonlinear flexural analysis of FGM plates subjected to patch loads. The effect of load distribution and geometric nonlinearity on deformation characteristics was brought out. Ahmadi et al. [34] analyzed the instability and buckling of bidirectional FG multiple nanobeam systems in the thermal environment using a meshless method and demonstrated its adaptability for nano-scale thermo-mechanical problems. Cui et al. [35]presented a smoothed Hermite radial point interpolation method (S-HRPIM) for the analysis of thin plates. Numerical stability and smooth variation of stresses were obtained without mesh dependence. Finally, Wang et al. [36] developed a new meshless method to solve the steady-state heat conduction problems in anisotropic and inhomogeneous media, demonstrating its versatility and accuracy in complex thermal analysis problems. Of these, the RBF-based collocation method is particularly found to be very reliable, with quicker convergence and better accuracy compared to conventional FE approaches. The RBF approximations are well-suited for the numerical modeling of skew plate and shell structures.

In this investigation, a collocation computational method employing RBF is developed by coupling the five-variable HSDT. The main novelty of the present work lies in the application of the RBF-based collocation method within the HSDT framework, specifically to skew plate geometries, which have received comparatively limited attention in existing studies. Unlike Refs. Ferreira et al. [29] and Ferreira et al.,[37] which primarily focus on regular plate configurations, the present study systematically investigates the influence of skew angle on vibration characteristics. Additionally, the study provides a detailed assessment of the effect of an elastic foundation for skew geometries, which is not explicitly addressed in the literature. The combination of skew plate analysis, parametric investigation, and implementation within the HSDT–RBF framework constitutes the key contribution of this work. The proposed model facilitates a thorough examination of the free vibration behavior of simply supported skew laminated plates under different influential parameters. This investigation will help in the efficient design and development of skew laminated plates with elastic foundations. The paper structure is as follows: Section 2 provides the theoretical background of skew laminated plates and the HSDT theory. Section 3 explains the vibration equation derivation by using RBF. Section 4 provides numerical simulations to confirm the proposed theory and investigates the influence of skewness on the vibration response of plates with elastically supported edges. Lastly, Section 5 summarizes the most important conclusions arrived at from this study.

2. Formulation Approach

A skew composite plate supported by an elastic foundation and laminated is considered, and its in-plane dimensions are represented as a and b along the global x- and y-directions, respectively. The reference plane for analysis is taken at the mid-surface of the plate and denoted at z=0. The plate is made up of a number of orthotropic layers with a particular fiber orientation, leading to material property variation in the thickness direction. These layers are considered to be ideally bonded, and this makes both transverse shear stresses and displacement components continuous across the interfaces. The geometry and coordinates of the plate are shown in Fig. 1.

Using the HSDT formulation, the displacement components u v and w of any point inside the plate are given as a function of the mid-surface displacements u_0, v_0 and w_0 and the ϕ_x, ϕ_y bending rotational components. The general displacement field thus captures both in-plane extension and bending, and higher-order variation of transverse shear deformation. The mathematical expression of the displacement field is of the general form presented by [38].

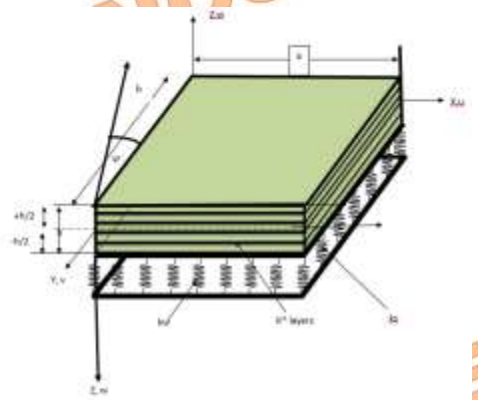


Fig. 1. Geometrical description of the skew laminated plate

$$
\begin{aligned}
u &= u_0(x,y) - z\frac{\partial w_0(x,y)}{\partial x} + g(z)\phi_x(x,y) \\
v &= v_0(x,y) - z\frac{\partial w_0(x,y)}{\partial y} + g(z)\phi_y(x,y) \\
w &= w_0(x,y)
\end{aligned} \tag{1}
$$

where, $g(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$

The strain in the plate comprises in-plane normal and shear strains together with transverse shear strains, and these are formulated as [39].

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
\end{aligned} \tag{2}
$$

The shear and normal strains are obtained by differentiating the displacement field. [40]:

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + g(z)\frac{\partial \phi_x}{\partial x} \\
\varepsilon_{yy} &= \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2} + g(z)\frac{\partial \phi_y}{\partial y} \\
\gamma_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} + g(z)\frac{\partial \phi_x}{\partial y} + g(z)\frac{\partial \phi_y}{\partial x} \\
\gamma_{xz} &= \frac{\partial g(z)}{\partial z}\phi_x \\
\gamma_{yz} &= \frac{\partial g(z)}{\partial z}\phi_y
\end{aligned} \tag{3}
$$

The stress-strain relationship using the global coordinate system is provided as [38]:

$$
\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}_k \right\} \tag{4}
$$

where the specifications $\bar{Q}_{ij}$ are transformed, reduced stiffness coefficients for the layer, and represented as:

$$
\begin{aligned}
\bar{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta.\cos^2\theta \\
&+ Q_{22}\sin^4\theta \\
\bar{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta.\cos^2\theta \\
&+ Q_{22}\cos^4\theta \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta.\cos^2\theta \\
&+ Q_{12}(\sin^4\theta + \cos^4\theta) \\
\bar{Q}_{16} &= (Q_{11} - Q_{22} - 2Q_{66})\sin\theta.\cos^3\theta \\
&+ (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta.\cos\theta \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta.\cos\theta \\
&+ (Q_{12} - Q_{22} + 2Q_{66})\sin\theta.\cos^3\theta \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})\sin^2\theta.\cos^2\theta \\
&+ Q_{66}(\sin^4\theta + \cos^4\theta) \\
\bar{Q}_{44} &= Q_{44}\cos^2\theta + Q_{55}\sin^2\theta \\
\bar{Q}_{45} &= (Q_{55} - Q_{44})\cos\theta.\sin\theta \\
\bar{Q}_{55} &= Q_{55}\cos^2\theta + Q_{44}\sin^2\theta
\end{aligned} \tag{5}
$$

Where,

$$
\begin{aligned}
&Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}};\ Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}; \\
&Q_{12} = \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}};\ Q_{44} = G_{23}; \\
&Q_{55} = G_{13};\ Q_{66} = G_{12};
\end{aligned} \tag{6}
$$

The GDEs for the laminated composite plate are obtained using Hamilton's principle, which is a variational approach for deriving the equations of motion. Mathematically, Hamilton's principle is given as [41]:

$$\int_{t_1}^{t_2} \delta(K-U-EF)\,dt = 0 \qquad (7)$$

where, K = Kinetic energy, U = Strain energy, EF= Elastic foundation

The equations for the system's kinetic energy (T) and for the system's strain energy (U) are derived as [40].

$$K=\frac{1}{2}\int_{Volume}\rho\left(\left(\frac{\partial u}{\partial\tau}\right)^2+\left(\frac{\partial v}{\partial\tau}\right)^2+\left(\frac{\partial w}{\partial\tau}\right)^2\right)dxdydz \qquad (8)$$

$$U=\frac{1}{2}\int_{Volume}\left(\sigma_{xx}\varepsilon_{xx}+\sigma_{yy}\varepsilon_{yy}+\sigma_{xy}\gamma_{xy}+\sigma_{yz}\gamma_{yz}+\sigma_{xz}\gamma_{xz}\right)dxdydz \qquad (9)$$

The stored energy in the elastic foundation, which is represented by, can be written as follows. [42].

$$EF=\frac{1}{2}\left(\int_A Kw(w_o^2)-Ks\left(\left(\frac{\partial w_o}{\partial x}\right)^2+\left(\frac{\partial w_o}{\partial y}\right)^2\right)dA\right) \qquad (10)$$

The GDEs of the plate are acquired by collecting the coefficients of δu1, δu2, δu3, φ_x and φ_y is formulated as [15]:

$$\delta u_1:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\left(M_0\frac{\partial^2 u_0}{\partial\tau^2}-M_1\frac{\partial^3 w_o}{\partial x\partial\tau^2}+M_3\frac{\partial^2\phi_x}{\partial\tau^2}\right) \qquad (11)$$

$$\delta u_2:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=\left(M_0\frac{\partial^2 v_o}{\partial\tau^2}-M_1\frac{\partial^3 w_o}{\partial y\partial\tau^2}+M_3\frac{\partial^2\phi_y}{\partial\tau^2}\right) \qquad (12)$$

$$\delta u_3:\ \frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}-K1(u_3)+K2\left(\frac{\partial^2 w_o}{\partial x^2}+\frac{\partial^2 w_o}{\partial y^2}\right)=$$
$$\left[\begin{array}{l} M_0\dfrac{\partial^2 u_3}{\partial\tau^2}+\left(\dfrac{\partial^3 u_1}{\partial x\partial\tau^2}+\dfrac{\partial^3 u_2}{\partial y\partial\tau^2}\right)M_1 \\ -\left(\dfrac{\partial^4 w_o}{\partial x^2\partial\tau^2}+\dfrac{\partial^4 w_o}{\partial y^2\partial\tau^2}\right)M_2+\left(\dfrac{\partial^4\phi_x}{\partial x^2\partial\tau^2}+\dfrac{\partial^4\phi_y}{\partial y^2\partial\tau^2}\right)M_3 \end{array}\right] \qquad (13)$$

$$\delta\phi_x:\ \frac{\partial M_x^f}{\partial x}+\frac{\partial M_{xy}^f}{\partial y}-S_x^f=\left(M_3\frac{\partial^2 u_{10}}{\partial\tau^2}-M_4\frac{\partial^3 w_0}{\partial x\partial\tau^2}+M_5\frac{\partial^2\phi_x}{\partial\tau^2}\right) \qquad (14)$$

$$\delta\phi_y:\ \frac{\partial M_{xy}^f}{\partial x}+\frac{\partial M_y^f}{\partial y}-S_y^f=\left(M_3\frac{\partial^2 v_o}{\partial\tau^2}-M_4\frac{\partial^3 w_o}{\partial y\partial\tau^2}+M_5\frac{\partial^2\phi_y}{\partial\tau^2}\right) \qquad (15)$$

The force and moment results on the plate are represented as [15]:

$$N_{ij},\,M_{ij},\,M_{ij}^f=\int_{-h/2}^{+h/2}\left(\sigma_{ij},\,z\sigma_{ij},\,g(z)\sigma_{ij}\right)dz \qquad (16)$$

$$S_x^f,\,S_y^f=\int_{-h/2}^{+h/2}\left(\sigma_{xz},\,\sigma_{yz}\right)\left(\frac{\partial g(z)}{\partial z}\right)dz \qquad (17)$$

The plate stiffness coefficients are represented as [15]:

$$A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\overline{C}_{ij}\left(1,z,z^2,g(z),z\,g(z),g^2(z)\right)dz \qquad (18)$$

i, j = 1, 2, 6

$$A_{ij}=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\overline{C}_{ij}\left(\frac{\partial g(z)}{\partial z}\right)^2 dz \qquad \text{i, j = 4, 5} \qquad (19)$$

$$M_0,M_1,M_2,M_3,M_4,M_5=\rho\int_{-h/2}^{+h/2}\left(1,z,z^2,g(z),zg(z),g^2(z)\right)dz \qquad (20)$$

The simply supported (SSSS) boundary conditions are taken as:

$$u_s,\phi_s,w_0,N_{nn},M_{nn}=0 \qquad (21)$$

where,

$$u_s=-n_y.u_o+n_x.v_0 \qquad (22)$$
$$\phi_s=-n_y.\phi_x+n_x.\phi_y$$
$$N_{nn}=n_x^2N_{xx}+2n_xn_yN_{xy}+n_y^2N_{yy}$$
$$M_{nn}=n_x^2M_{xx}+2n_xn_yM_{xy}+n_y^2M_{yy}$$
$$n_x=\cos(\psi),\quad n_y=\sin(\psi)$$

3. Methodological Approach

The solution strategy in the analysis of free vibration behavior of skew laminated plates employing the MQRBF method with a strong form collocation technique includes the discretization of the governing equations obtained from HSDT directly in physical space. The displacement field is first formulated taking into account the geometric skewness and laminated stacking sequence. The strong form of GDEs is formulated based on Hamilton's principle, including boundary conditions and initial assumptions. MQRBF is utilized to represent the unknown field variables, providing smooth and accurate approximations without the necessity of a predetermined mesh. Collocation points are located strategically throughout the domain, such as near boundaries, and the differential operators are directly applied at these locations to impose the governing equations and boundary conditions. The collocation approach manages complex geometries such as skew plates efficiently and captures the vibration response via eigenvalue analysis, making precise predictions of natural frequencies without needing numerical integration or element connectivity.

The expression for MQRBF is $g=\left(r^2+c^2\right)^{0.5}$;

where 'c' is the shape parameter. n_x and n_y are the number of divisions along the 'a' and 'b' axes, respectively. α is a constant that governs the value of 'c' for interior and boundary nodes for MQRBF. Choosing an appropriate shape parameter remains a challenging issue for researchers, as both the accuracy and numerical stability of the solution are highly sensitive to its value. It is well recognized that achieving optimal stability and accuracy simultaneously is difficult. In the present study, a MATLAB code has been developed to obtain solutions based on the proposed formulation. A convergence study was first carried out by systematically varying the shape parameter. After ensuring stable convergence, the accuracy of the results was evaluated. Based on this combined assessment of stability and accuracy, an optimal value of the shape parameter 'c' is proposed, which is expressed as:

$$c = \alpha.\sqrt{\left(\frac{a}{n_x}\right)^2 + \left(\frac{b}{n_y}\right)^2} \tag{23}$$

Any variable u0 can be interpolated in the form of the radial distance between nodes using the following expression:

$$u_1 = \sum_{j=1}^{N} \alpha_j^{u_1} f\left(\left\| X - X_j \right\|\right) \tag{24}$$

where N is the total number of nodes, α_j^u is an unknown coefficient.

The solution of GDEs by MQRBF considers domain and boundary nodes. The unknown field variables uo, vo, wo, ϕ_x and ϕ_y appearing in GDEs are assumed to be[15]:

$$u_o(x,y) = \sum_{j=1}^{N} \alpha_j^{u_o} f\left(\left\| X - X_j \right\|\right) \tag{25}$$

$$v_o(x,y) = \sum_{j=1}^{N} \alpha_j^{v_o} f\left(\left\| X - X_j \right\|\right) \tag{26}$$

$$w_o(x,y) = \sum_{j=1}^{N} \alpha_j^{w_o} f\left(\left\| X - X_j \right\|\right) \tag{27}$$

$$\phi_x(x,y) = \sum_{j=1}^{N} \alpha_j^{\phi_x} f\left(\left\| X - X_j \right\|\right) \tag{28}$$

$$\phi_y(x,y) = \sum_{j=1}^{N} \alpha_j^{\phi_y} f\left(\left\| X - X_j \right\|\right) \tag{29}$$

For the natural frequency issue, assume a harmonic solution in displacements as[15]:

$$u_o(x,y,t) = u_o(x,y)e^{i\omega t} \tag{30}$$

$$v_o(x,y,t) = v_o(x,y)e^{i\omega t} \tag{31}$$

$$w_o(x,y,t) = w_o(x,y)e^{i\omega t} \tag{32}$$

$$\phi_x(x,y,t) = \phi_x(x,y)e^{i\omega t} \tag{33}$$

$$\phi_y(x,y,t) = \phi_y(x,y)e^{i\omega t} \tag{34}$$

The GDEs and boundary conditions can be written in the following manner.

$$\left[[K]_L + [M]\right]\{\delta\} = 0 \tag{35}$$

Where,

$$\left[\begin{bmatrix}[K]_L \\ [K]_B\end{bmatrix}_{5N\times5N} + \omega^2 \begin{bmatrix}[M] \\ 0\end{bmatrix}_{5N\times5N}\right]\{\delta\}_{5N\times1} = 0 \tag{36}$$

The methodology of the solution presented herein is coded in MATLAB 2019. All the numerical calculations are done via MATLAB on a system with a 2.7 GHz Intel Core i7 processor. To ensure the accuracy and reliability of the suggested solution approach, an exhaustive convergence study is performed and presented in the next section.

4. Results and Interpretation

Using the five variables HSDT's mathematical model and solution techniques based on MQRBF, a generalized computational work in the form of code in MATLAB (2019). Several examples are considered for the free vibration analysis of skew laminated plates. Convergence studies are first investigated to check the efficiency and consistency of the present technique and mathematical model. The natural frequency is normalized as [13]

$$\varpi = \frac{\omega a^2}{h}\sqrt{\frac{\rho}{E_2}} \tag{37}$$

Table 1: Convergence assessment for frequency parameter of simply supported skew laminated plate with various skew angles (E1/E2=40) a/h=10

Source	Skew Angle			
	0	15	45	60
Liew et al.[13]	1.5709	1.6886	2.8798	4.4998
Wang,[43]	1.5699	-----------	2.8825	-------
Ferreira et al.,[37]	1.5791	1.6917	2.8228	4.3761
Present method (7x7 nodes)	1.4758	1.7856	2.6089	4.3201
Present method (9x9 nodes)	1.5573	1.7335	2.7640	4.5697
Present method (11x11 nodes)	1.5696	1.7139	2.775	4.5760
Present method (13x13 nodes)	1.5743	1.7062	2.8378	4.5478

Present method (15x15 nodes)	1.5711	1.72	2.8553	4.5229

Where ρ is density.

The properties of the materials used in computations are

E1/E2=varies, G12 = G13= 0.5E2, G23=0.2E2, ν12=0.25:

4.1. Convergence Assessment and Comparative Study

For the accuracy, correctness, and consistency of the collocation approach, convergence and validation studies have been conducted. Convergence analysis ensures the reliability and stability of the computational results by node distribution, whereas validation provides a comparison of the derived results with available benchmark solutions or published data to ascertain the accuracy of the proposed model.

Table 1 presents a comprehensive convergence study of the normalized natural frequency of a skew laminated composite plate with the lay-up sequence of (90/0/90/0). The analysis is performed for the simply supported square plate with a/h=10 and E1/E2=40. The obtained results evidently prove that the normalized natural frequencies show satisfactory convergence characteristics with minimal variation recorded beyond a discretization level of 13×13 nodes. This verifies the sufficiency of the used node size for reliable prediction of the frequency. In addition, the results obtained here are compared with known results documented in the literature through the works of Liew et al. [11], Wang [12], and Ferreira et al. [13], who used other higher-order and advanced numerical methods. The present numerical frequencies agree very well with the published data, further proving both the accuracy and consistency of the used formulation. This convergence shows the ability of the proposed model to analyze skew laminated composite plates with intricate lay-up configurations. Table 2 shows the convergence study of the first five modes of normalized natural frequency of a five-layered skew laminated composite plate [45/-45/45/-45/45], computed for skew angles 0° and 30°, with an elastic modulus ratio E1/E2 =40. The analysis examines the influence of nodes on the accuracy of the frequency predictions for various vibration modes. The findings show that all five normalized natural modes for the laminated plates come together quickly, with little change apparent beyond a node with 13×13 nodes for both skew angles. This demonstrates the numerical stability and accuracy of the formulated method in obtaining skewed laminated plate configurations. To confirm the precision of the present method with the HSDT model, the findings have been compared against available solutions published by Wang [14] and Anlas and Goker [15]. The comparison indicates a very good correlation between the current results and the reference data, further affirming the validity of the numerical method used. According to the results of different convergence and validation experiments conducted in Tables 1 to 3, it is concluded that the node discretization of 15×15 nodes is an optimal value between computation and solution accuracy. Therefore, it is employed for the rest of the calculations concerning the vibration characteristics of laminated composite plates.

Table 1. : Validation study for the normalized natural frequency of a simply supported skew laminated [45/-45/45/-45/45] plate with five nodes (E1/E2=40).

Skew angle	Source	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
0°	Wang [44]	1.9141	3.9745	6.6567	7.6564	8.1511
	Anlas and Goker [45]	1.9100	3.980	6.66	7.66	8.15
	Present method (7x7 nodes)	1.8306	4.2762	6.6097	8.5287	9.1335
	Present method (9x9 nodes)	1.8741	4.0661	6.4779	7.7864	8.1132
	Present method (11x11 nodes)	1.8855	3.9894	6.4342	7.8255	7.9823
	Present method (13x13 nodes)	1.8898	3.955	6.4158	7.6484	7.9263
	Present method (15x15 nodes)	1.8918	3.9372	6.4068	7.5534	7.8974
30°	Wang [44]	2.8248	5.1819	8.4836	9.2574	12.107
	Anlas and Goker [45]	2.84	5.2	8.49	9.29	12.18
	Present method (7x7 nodes)	2.9238	5.2939	8.5856	9.2736	13.4059
	Present method (9x9 nodes)	2.8684	5.1883	8.1229	8.2994	11.6545
	Present method (11x11 nodes)	2.8414	5.1387	8.1907	8.9336	11.9027
	Present method (13x13 nodes)	2.8258	5.1118	8.1397	8.8742	11.6954
	Present method (15x15 nodes)	2.8153	5.0955	8.113	8.8383	11.5848

4.3. Parametric Analysis

Table 3 gives the normalized natural frequency values of a four-layered skew laminated composite plate (E1/E2=25, a/h =10, a=b, kw =10, ks=5) with different fiber orientation schemes. The research assesses the vibrational behavior for the following laminate configurations: (90/0/0/90), (0/90/90/0), (30/–30/–30/30), and (75/–75/–75/75). Based on the values demonstrated in the table, the normalized natural frequency is seen to rise systematically as the skew angle in the plate increases, irrespective of the fibre orientation scheme. This behavior arises due to the increase in the effective stiffness of the plate due to geometric transformation caused by skewness. The greater the skew angle, the stronger the structural rigidity of the plate, presenting more resistance to deformation, and hence higher natural frequencies. Surprisingly, this growing trend of improvement is noted in all the evaluated patterns of fiber orientation, which reveals that the inclining effect of the skew angle prevails and is favorably contributing to the vibration stiffness enhancement of laminated plates.

Table 3: Effect of fiber orientation schemes on normalized natural frequency of the laminated skew plate.

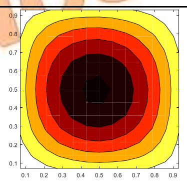
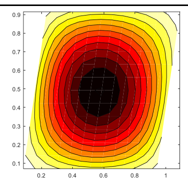
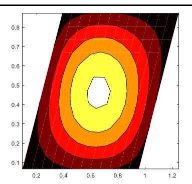
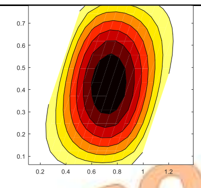
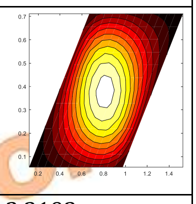
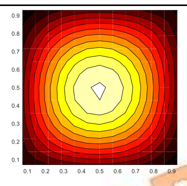
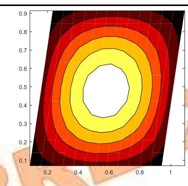
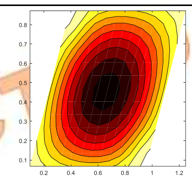
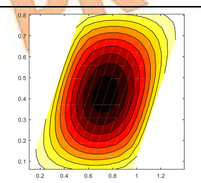
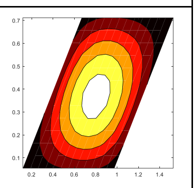
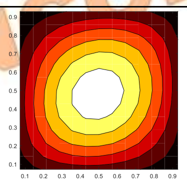
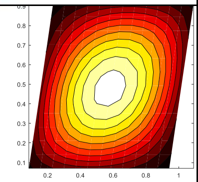
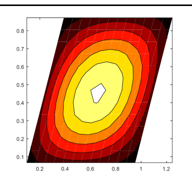
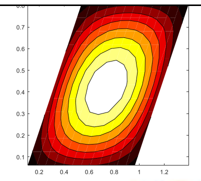
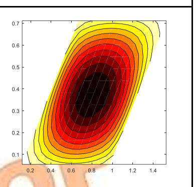
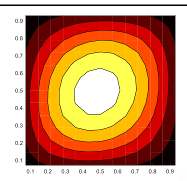
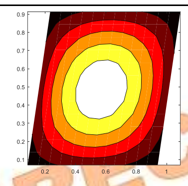
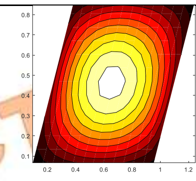
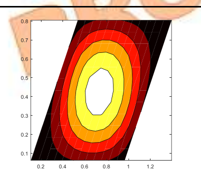
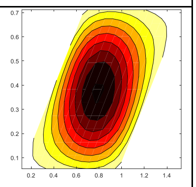
Orientation	Skew Angle				
	0	10	20	30	40
(90/0/0/90)					
	1.6932	1.7387	1.8866	2.1757	2.3193
(0/90/90/0)					
	1.6932	1.7003	1.8187	1.8608	2.1630
(30/-30/-30/30)					
	1.6673	1.7945	1.7976	1.8211	1.9018
(75/-75/-75/75)					
	1.7225	1.7244	1.7795	2.0927	2.4012

Table 5: Influence of the elastic foundation parameters on the normalized natural frequency of a skew laminated plate under different fiber orientation schemes

Orientation	(kw, ks)									
	(0,0)	(10,0)	(20,0)	(50,0)	(0,10)	(0,20)	(0,50)	(10,10)	(20,20)	(50,50)
90/0/0/90	1.7042	2.3352	2.8268	3.2112	1.7336	1.7624	1.8464	2.3567	2.8624	3.2112
0/90/90/0	1.3892	2.1194	2.6531	3.2112	1.4250	1.4599	1.5601	2.1431	2.6910	3.2112
30/-30/-30/30	1.2935	2.0864	2.6387	3.8282	1.3318	1.3690	1.4751	2.1104	2.6767	3.8937

75/-75/-75/75	1.6388	2.2773	2.7747	3.9058	1.6693	1.6992	1.7860	2.2994	2.8108	3.9700

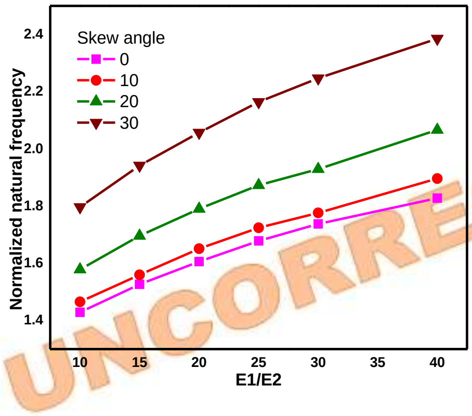


Fig 2 Effect of E1/E2 on the normalized natural frequency of elastically supported laminated skew plate.

Figure 2 demonstrates the impact of different orthotropic stiffness ratios on the normalized fundamental frequency of a four-layered simply supported skew laminated composite plate (a/h=10, kw =5, ks =5) with lay-up sequence (90/0/0/90). The parameters under consideration are

It can be easily seen that as the orthotropic stiffness ratio increases, the natural frequency also increases. This is due to the increase in in-plane stiffness in the fiber direction. A greater ratio indicates a stiffer material in the principal fiber direction, making the laminated structure more rigid. When the skew angle of the laminated plate is increased, the normalized natural frequency is also raised. This is largely because of the geometric transformation effect generated through skewness, which acts to improve the overall structural stiffness of the plate. With an increase in stiffness, there is increased resistance against vibrational deformation, and hence the result is higher natural frequencies. Table 4 presents the combined effect of a/h and skew angle on the normalized natural frequency of a four-layered laminated composite plate with a lay-up sequence of (30/−30/−30/30). The analysis reveals two key trends. First, the normalized natural frequency increases from thick to thin plate. However, it is also observed that the influence of a/h>50 indicates an asymptotic trend where further increases in slenderness do not significantly alter the vibrational characteristics. Second, the normalized natural frequency also rises with an increase in the skew angle. This is due to geometric stiffening induced by the skewed configuration of the plate. By increasing the skew angle, the radial distances between the two adjacent nodes near the corner of connecting boundaries decrease significantly as compared to other parts of the geometry, causing one element of the stiffness matrix to be very small as compared to other parts of the stiffness matrix. This makes the matrix ill-conditioned if inverted. For this reason, the solution becomes unstable and inconsistent. This is the main reason for limiting the skew angle. Even this problem occurs if we use other methods, like FEM, also.

Table 4: Effect of span to thickness ratio on natural frequency of skew laminate plate

a/h	Skew angle			
	0	15	30	60
10	1.351	1.4764	1.7018	2.079
20	1.5046	1.7046	2.1839	4.1579
30	1.545	1.7647	2.291	6.2369
40	1.5601	1.7888	2.3539	6.657
50	1.5673	1.8009	2.3879	6.7924
100	1.577	1.577	2.4302	6.989

Table 5 gives the effect of elastic foundation parameters on the normalized natural frequency for a 30° skew laminated plate with a skew angle of 30°, a thickness ratio a/h=10, and a material stiffness ratio E1/E2=20. The study examines different fiber orientation patterns to evaluate the influence of the foundation's stiffness coefficients, usually represented by Winkler and shear layer parameters, on the free vibration response of the plate. Findings show that a higher value for either kw or ks results in an increase in the normalized natural frequency, which indicates the stiffening effect of the elastic foundation. The effect of ks is more sensitive than that of kw.

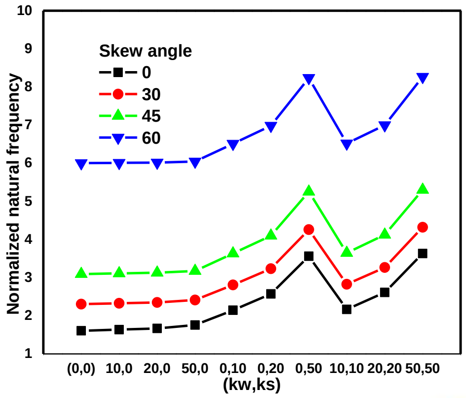
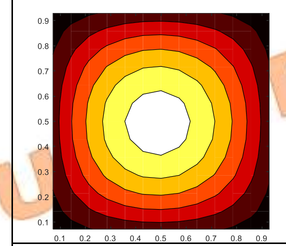
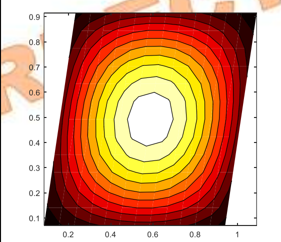
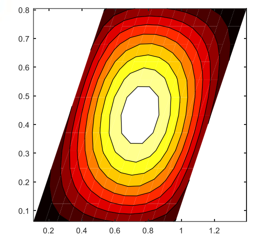
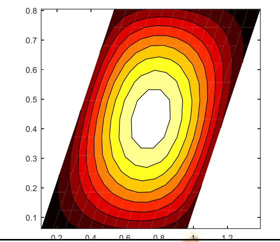
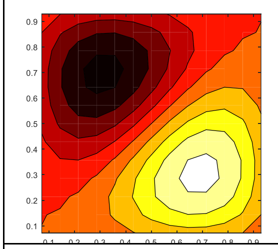
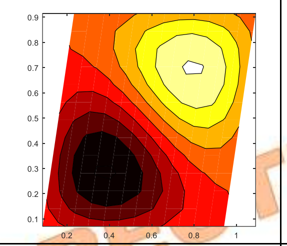
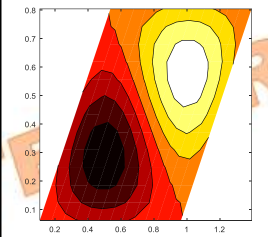
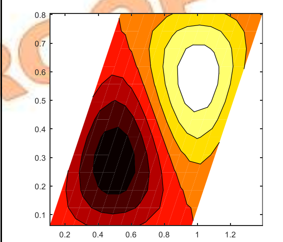
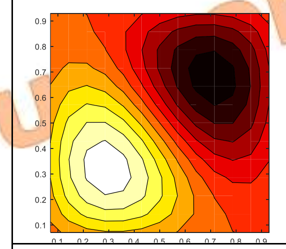
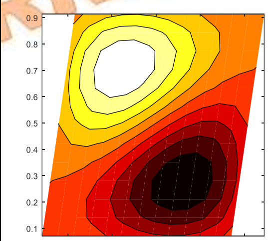
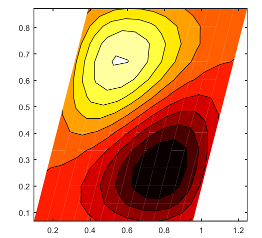
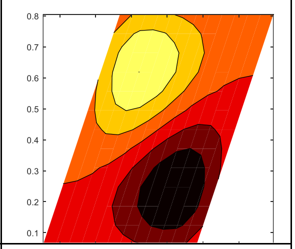


Fig 3: Effect of skew angle and elastic foundation parameters on the normalized natural frequency of a five-layer (90/0/90/0/90) skew laminated plate.

Figure 3 shows the influence of skew angles and elastic foundation parameters on the normalized natural frequency of a five-layer (90/0/90/0/90) skew laminated plate with a thickness ratio a/h=20 and material property ratio. It is noted that as the foundation stiffness parameters, kw and ks, increase, the normalized natural frequency also increases. Nonetheless, the impact of ks is stronger than that of kw, which means shear interactions between the plate and the base play a stronger role in affecting the vibration response. Also, the higher the skew angle, the higher the normalized natural frequency becomes. This is due to the effect of geometric stiffening caused by skewness that increases the stiffness of the laminated plate. Table 6 illustrates the influence of skew angles on the contour patterns of the first three vibration modes corresponding to the normalized natural frequency of an elastically supported five-layer (90/0/90/0/90) skew laminated plate, with geometric ratio a/h=10 and foundation parameters kw =5 and ks =2. The contour plots clearly depict the modal deformation characteristics for each mode shape at different skew angles. It is observed that, for all considered skew angles, smooth and well-defined contour patterns are obtained, indicating the numerical stability and consistency of the adopted approach. As the skew angle increases, a noticeable change in the modal distribution can be seen, with contours becoming more skewed and asymmetric in accordance with the geometry of the plate. Moreover, an increasing trend in the normalized natural frequency is observed with the increase in skew angle for all three modes.

Table 6: Effect of skewness for the three modes contour on the normalized natural frequency of elastically supported laminated

Mode	Skew angle			
	0	10	20	30
I	 1.4240	 1.4759	 1.6050	 1.8809
II	 2.3990	 2.5285	 2.8216	 3.0963
III	 2.3990	 2.7576	 2.9990	 3.2680

5. Conclusions

This study presents an efficient MQRBFC for analyzing the free vibration behavior of simply supported laminated skew plates resting on elastic foundations. By employing a five-variable HSDT and formulating the governing equations through Hamilton's principle, the proposed method accurately captures the structural response without the need for shear correction factors. The numerical implementation developed in MATLAB successfully evaluates the natural frequencies of skew laminated plates under various geometric and material conditions. Results show that in thicker plates, foundation stiffness has a greater effect, whereas in thin plates, the skew angle has a primary influence on vibration behavior. The orientation of fibers also significantly affects the fundamental frequency, especially at high skew angles. A rise in skew angle results in a significant increase in natural frequencies owing to the stiffening effect due to geometry, particularly in lower vibration modes. The occurrence of elastic foundation parameters also increases the frequency response by raising the overall stiffness of the plate system. The normalized vibration frequencies are also found to increase with an increased span-to-thickness ratio (a/h), and the normalized fundamental frequency is found to increase as the orthotropy ratio increases. The study further emphasizes that the parameters of orthotropic stiffness ratio, skew angle, thickness ratio, and foundation characteristics play a key role in determining the dynamic response. The use of HSDT improves the precision of the displacement field by considering a realistic parabolic shear strain distribution, thus removing the necessity for shear correction factors. Subsequent research can apply this approach to other laminated composite structures, including shells and truss-type structures, to advance model accuracy and application. The present study is limited to simply supported edges, and extensions to other boundary conditions are being pursued separately by associated research groups and are therefore not included here for ethical reasons.

The proposed formulation is general and can be extended to more complex structural configurations such as functionally graded structures, sandwich plates, and shell geometries by incorporating suitable material gradation and curvature effects. Moreover, its compatibility with higher-order and meshless computational approaches makes it suitable for analyzing large-scale and heterogeneous structural systems for buckling and flexure analysis, also in future studies.

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Conflicts of Interest

The authors declare that they have no conflict of interest.

Contributions

A.K. Singh: Conceptualization, Data collection, Analysis, Methodology, Literature review, Model development. J. Singh: Supervision, Writing – review & editing. All authors have read and approved the final manuscript.

Data availability statement

Present both original data collected during your research as well as any secondary reuse of existing data that informs your results and analyses.

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