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# Optimization of Hybrid Composite Laminate Based on the Frequency using Imperialist Competitive Algorithm

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**ABSTRACT**

Imperialist competitive algorithm (ICA) is a new socio-politically motivated global search strategy. The ICA is applied to hybrid composite laminates to obtain minimum weight and cost. The approach which is chosen for conducting the multi-objective optimization was the weighted sum method (WSM). The hybrid composite Laminates are made of glass/epoxy and carbon/epoxy to combine the lightness and economical attributes of the first with high-stiffness property of the second in order to make trade-off between the cost and weight as the objective functions and natural flexural frequency as a constraint. The results were evaluated for different weighting factors ($\alpha$) including optimum stacking sequences, and number of plies made of either glass or carbon fibers using the ICA, and were compared with those using the genetic algorithm (GA) and ant colony system (ACS). The comparisons confirmed the advantages of hybridization and revealed that the ICA outperformed the GA and ACS in terms of function's value and constraint accuracy.



## 1. Introduction

Laminated composite materials find a wide range of applications in structural design, particularly in the field of automotive, aerospace and marine engineering. This is primarily due to the high specific strength and stiffness values with minimum weight that these type of materials offer.

The design of a structural component using composites involves both material and structural design. Laminated composites are usually designed due to the designer's needs by choosing the thickness, orientation and number of lamina. The thickness and orientation of the lamina are usually limited to some set values due to manufacturing limitations. Searching for the optimum solution in laminated composite structures is a discrete optimizing problem [1, 2].

In all applications, it is ideal to have the stiffest, lightest, and the most economical structures [3]. These three requirements normally act against each other and may come in compromise with the help of hybridization of composite laminates in which the high-stiffness material, generally more expensive and heavier, is used in the outer layers to provide enough rigidity and stiffness [4, 5]. The material used in the inner layers should bear lesser cost, be lighter, and have low-stiffness. Deflection, stress, and natural frequencies are some supplementary aspects which have been investigated in hybrid laminates in a multi-objective optimization process [6-8]. Javidrad and Nouri [9] applied a modified simu-

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lating annealing to minimize a cost function composed of the difference between the effective stiffness properties and weight of the considered laminate from the number of layers and the fiber angle of each layer.

Dynamic properties of composites are analysed for investigating structural issues [10-12] and optimal designs of composites with frequency objective or constraint which are carried out extensively [13, 14].

Single-objective maximization of the fundamental frequency for laminated plates was given by using continuous design variables [15-17]. The same design for cross-ply laminates was studied by Duffy and Adali [18] and for anisotropic laminates by Adali [19]. Apalak et al. [20] carried out the layer optimization for maximum fundamental natural frequency of composite laminates by a genetic algorithm.

Fukunaga et al. optimized the composite structures under natural frequency constraints, where only the thickness of each lamina is taken as the design variable [21]. Also, the optimization algorithm of laminate frequency issue has been investigated by Narita [22]; Narita and Hodgkinson [23]. Minimum cost design of laminated plates undergoing free vibrations was conducted by Adali and Duffy [24]. Farshi et al. presented a method based on the Ritz approximations for minimum thickness. multilayer rectangular composite laminates is presented which is based on a layerwise optimizing procedure under natural frequency limitations [25].

Adali and Verijenco [26] optimized stacking sequence design of symmetric hybrid laminates undergoing free vibrations for fundamental frequency and frequency separation.

Regarding multi-objective optimization, Tahani et al. optimized the fundamental frequency and cost using the genetic algorithm (GA) [27], and Kolahan et al. also solved the same problem with the help of simulated annealing (SA) [28]. Reliability based optimization of composite laminates for frequency constraint was investigated by Hao et al. [29].

Hemmatian et al. [5] and Grosset et al. [30] optimized the number and angle of glass/epoxy and carbon/epoxy layers in order to get minimal cost and weight subject to the first natural frequency by using elitist ant system and GA, respectively. Abachizadeh and Tahani applied ACS on hybrid laminate composite for minimum cost and weight and natural frequency as a constraint [31].

Recently a new meta-heuristic algorithm, so called imperialist competitive algorithm (ICA) is proposed by Atashpaz et al. [32, 33]. ICA is a socio-politically motivated optimization algorithm which is similar to many other evolutionary algorithms, and starts with a random initial population or empires. Each individual agent of an empire is called country and the countries are categorized into two types; colony and imperialist state that collectively form empires.

ICA is applied to structural optimum design by the Kaveh and Talatahari [34, 35]. Abdi et al. used ICA to find the optimal design of laminated composite structures due to the various failure criteria [36]. Also ICA has been developed for optimum design of composite plates based on weight and cost by Mozafari et al. [37]. Composite plates under thermal buckling loads are optimized using ICA [38]. Thermal buckling loads of laminated composite plates are maximized for a given total thickness. Esmaeilzadeh modified the empire movement toward the superior empire for balancing the exploration and exploitation abilities of the ICA [39].

In this study, weight and cost of symmetric balanced hybrid laminates were optimized considering the first natural flexural frequency as the design constraint. Results were compared with those obtained using GA [30] and ACS [31].

## 2. Analysis of Fundamental Flexural Frequency

Consider a simply supported symmetric hybrid laminated plate with the length a, width b, and total thickness h in the x, y, and z directions, respectively, as is shown in Fig. 1. Each layer has the thickness t and is idealized as a homogeneous orthotropic material. The total thickness of the laminate is equal to h=N×t which N is the total number of layers.

The hybrid laminate is made of $N_i$ inner and $N_o$ outer layers so that $N=N_i+N_o$. The governing equation of motion within the classical laminated plate theory for the described symmetric laminate is given as follows [40]:

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+D_{66})\frac{\partial^4 w}{\partial x^2 y^2} \\ +4D_{26}\frac{\partial^4}{\partial x\,\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}=\rho h\frac{\partial^2 w}{\partial t^2} \quad (1)$$

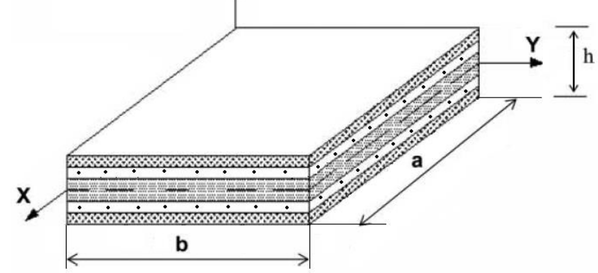



**Figure 1.** Geometry of composite laminate.

where $w$ is the deflection in the $z$ direction, $h$ is the total thickness, and $\rho$ is the mass density averaged in the thickness direction which is given by:

$$\rho = h^{-1}\int_{-h/2}^{h/2}\rho^{(k)}dz = \frac{1}{N}\sum_{k=1}^{N}\rho^{(k)} \tag{2}$$

where $\rho^{(k)}$ denotes the mass density of material in the $k^{th}$ layer. The bending stiffnesses $D_{ij}$ in Eq. (1) are defined as:

$$D_{ij} = \sum_{k=1}^{N}\int_{z_k}^{z_{k-1}}\bar{Q}_{ij}^{(k)}z^2 dz \tag{3}$$

where $\bar{Q}_{ij}^{(k)}$ is the transformed reduced stiffness of the $k^{th}$ layer [5]. A general form of solution for $w$ in the natural vibration mode *(m, n)* is presented as:

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{i\omega_{mn}t} \tag{4}$$

where $\omega_{mn}$ is the natural flexural frequency of the vibration mode *(m, n)* and $i=\sqrt{-1}$. *Substituting* Eq. (4) into Eq. (1) yields:

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h}\left[D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\right] \tag{5}$$

Different mode shapes are obtained by inserting different values of $m$ and $n$ where for the fundamental flexural frequency, both are put equal to one. Finally, the fundamental frequency (first frequency) is given as [5]:

$$f = \frac{\pi}{2\sqrt{\rho h}}\sqrt{\frac{D_{11}}{a^4} + \frac{2(D_{12}+2D_{66})}{a^2b^2} + \frac{D_{22}}{b^4}} \tag{6}$$

## 3. Imperialist Competitive Algorithm

Imperialist competitive algorithm (ICA) is a new socio-politically motivated global search strategy that has recently been introduced for dealing with different optimizing tasks [32]. This evolutionary optimizing strategy has shown great performance in both convergence rate and better global optima [41-44].

Like other evolutionary ones, the proposed algorithm starts with an initial population (countries in the world). Some of the best countries in the population are selected to be the *imperialists* and the rest form the *colonies* of these imperialists. All the colonies of initial population are divided among the mentioned imperialists based on their power. The power of an empire which is the counterpart of the fitness value in GA, is inversely proportional to its cost. After dividing all colonies among imperialists, these colonies start moving toward their relevant imperialist country. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. Then the imperialistic competition begins among all the empires. Any empire that is not able to succeed in this competition and can't/can not increase its power (or at least prevent decreasing its power) will be eliminated from the competition. Weak empires will lose their power and ultimately they will collapse. The movement of colonies toward their relevant imperialists along with competition among empires and also the collapse mechanism will hopefully cause all the countries to converge to a state in which there exists just one empire in the world and all the other countries are colonies of that empire. In this ideal new world, colonies have the same position and power as the imperialist [32].

### *3.1. Creation of Initial Empires*

An array of variable values to be optimized is generated. In the GA, this array is called "*chromosome*", but in ICA the term "*country*" is used for this array. In an $N_{var}$-dimentional optimisation problem, a country is a $1\times N_{var}$ array. This array is defined as following

$$country = [p_1, p_2, p_3, \ldots, p_{N_{var}}] \tag{7}$$

where $p_i$ are the variables to be optimized. The variable values in the country are represented as floating point numbers. The cost of a country is found by evaluation of the cost function $f$ at variables:

$$cost = f(country) = f(p_1, p_2, p_3, \ldots, p_{N_{var}}) \tag{8}$$

To start the optimisation algorithm, initial countries of size $N_{Country}$ is produced. We select $N_{imp}$ of the most powerful countries to form the empires. The remaining $N_{col}$ of the initial countries will be the colonies each of which belongs to an empire. To form the initial empires, the colonies are divided among imperialists based on their power. That is, the initial number of colonies of an empire should be directly proportionate to its power. To proportionally divide the colonies among imperialists, the normalized cost of an imperialist is defined by

$$C_n = c_n - \max_i\{c_i\} \tag{9}$$

where $c_n$ is the cost of the $n^{th}$ imperialist and $C_n$ is its normalized cost. Having the normalized cost of all imperialists, the normalized power of each imperialist is defined by

$$p_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \quad (10)$$

The initial colonies are divided among empires based on their power. Then the initial number of colonies of the $n^{th}$ empire will be

$$N.C._n = round\{p_n . N_{col}\} \quad (11)$$

where N.C.n is the initial number of colonies of the nth empire and Ncol is the total number of initial colonies. To divide the colonies, N.C.n of the colonies are randomly chosen and given to the nth imperialist. These colonies along with the nth imperialist form the nth empire. The bigger empires have greater number of colonies while weaker ones have lesser.

### 3.2. Moving the Colonies of an Empire toward the Imperialist

Movement of colonies toward their relevant imperialist is shown in Fig. 2. The colony moves toward the imperialist by movement vector where $x$ and $\theta$ are random variables with uniformed (or any proper) distribution:

$$x \sim U\left(0, \beta \times d\right) \quad (12)$$

$$\theta \sim U\left(-\gamma, \gamma\right) \quad (13)$$

where $\beta$ is a number greater than 1 and $d$ is the distance between the colony and the imperialist state. $\gamma$ is a parameter that adjusts the deviation from the original direction.

### 3.3. Revolution; A Sudden Change in Socio-Political Characteristics of a Country

Revolution is a fundamental change in power or organizational structures that takes place in a relatively short period of time that the colony randomly changes its position in the socio-political axis. The revolution rate in the algorithm indicates the percentage of colonies in each colony which will randomly change their position.

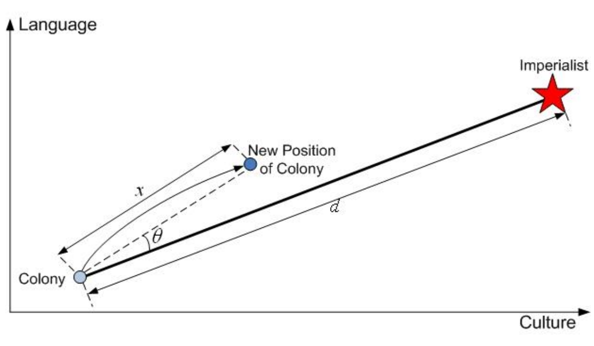



**Figure 2.** Movement of colonies toward their relevant imperialist in a randomly deviated direction [32]

### 3.4. Exchanging Positions of the Imperialist and a Colony

While moving toward the imperialist, a colony might reach to a position with lower cost than the imperialist. In this case, the imperialist and the colony change their positions. Then the algorithm will continue by the imperialist in the new position and the colonies will be assimilated by the imperialist in its new position.

### 3.5. Total Power of an Empire

Total power of an empire is mainly affected by the power of imperialist country. However the power of the colonies of an empire has an effect, albeit negligible, on the total power of that empire. This fact is modelled by defining the total cost of an empire:

$$\begin{aligned} T.C. &= Cost\,(imprialist_n) \\ &+ \xi mean\{ Cost(colonies\ of\ \ empire_n)\} \end{aligned} \quad (14)$$

Where T.C.n is the total cost of the nth empire and ξ is a positive small number.

### 3.6. Total Imperialistic Competition

All empires try to take the possession of colonies of other empires and control them. The imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. To start the competition, first a colony of the weakest empires is chosen and then the possession probability of each empire is found. The possession probability $P_P$ is proportionate to the total power of the empire. The normalized total cost of an empire is simply obtained by

$$N.T.C._n = T.C._n - \max_i\{T.C._i\} \quad (15)$$

Where, $T.C._n$ and $N.T.C_n$ are the total cost and the normalized total cost of $n^{th}$ empire, respectively. Having the normalized total cost, the possession probability of each empire is given by

$$p_{p_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right| \quad (16)$$

To divide the mentioned colonies among empires vector P is formed as following:

$$\mathrm{P} = \left[ p_{p_1}, p_{p_2}, p_{p_3}, \ldots, p_{p_{N_{imp}}} \right] \quad (17)$$

Then the vector R with the same size as P whose elements are uniformly distributed random numbers is created,

$$\mathrm{R} = \left[ r_1, r_2, r_3, \ldots, r_{N_{imp}} \right] \quad \text{and } r_1, r_2, r_3, \ldots, r_{N_{imp}} \approx U\,(0,1) \tag{18}$$

Then vector D is formed by subtracting R from P

$$\mathrm{D} = \mathrm{P} - \mathrm{R} = \left[ D_1, D_2, D_3, \ldots, D_{N_{imp}} \right] = \left[ p_{p_1} - r_1,\ p_{p_2} - r_2,\ p_{p_3} - r_3\ , \ldots, p_{p_{N_{imp}}} - r_{N_{imp}} \right] \tag{19}$$

Referring to vector D the mentioned colony (colonies) is handed to an empire whose relevant index in D is maximized.

### *3.7. The Powerless Empires*

Powerless empires will collapse in the imperialistic competition and their colonies will be divided among other empires. An empire collapses when it loses all of its colonies.

### *3.8. Convergence*

After a while all the empires except the most powerful one will collapse and all the colonies will be under the control of this unique empire. In this ideal new world all the colonies will have the same positions and same costs and they will be controlled by an imperialist with the same position and cost as themselves. In such a condition, the algorithm will be stopped. The Fig. 3 shows the flowchart of the proposed algorithm.

## 4. Problem Description

The design problem here is the selection of the optimal stacking sequence of hybrid composite laminate to obtain the simultaneous minimization of the weight and cost of a rectangular plate with the length *a=0.9144 m*, width *b=0.762 m,* and layer thickness *t=0.127 mm* subjected to a constraint on the first natural flexural frequency having lower bound of *25* Hz. This frequency is calculated based on the formulations presented in section 2 (Eq. (6)). The concept of hybridization is studied by using a two-material composite in which the lightness and high-stiffness carbon/epoxy layers and inexpensive low-stiffness glass/epoxy layers are considered. In this way, beside providing suitable structural rigidity, cost reduction which is always a significant and worthy goal can be achieved. The stiffness-to-weight ratio of carbon/epoxy is about four times higher than that of glass/epoxy with $E/\rho$=0.087 against $E/\rho$=0.022. However, it is also more expensive, with a cost per kilogram that is 8 times higher than that of glass/epoxy. If the higher priority is based on the weight, then, carbon/epoxy will be preferred. But if the cost is the paramount, the optimum laminate will contain glass/epoxy plies. The design of this simple rectangular plate leads us to study the trade-off between these two objective functions. The problem was investigated here, as mentioned in Section 1, is taken from Grosset et al. [30] and Abachizadeh and Tahani [31] and the results are compared with those obtained. The fiber orientation may take any value from a set of 19 angles ranging from $0^o$ to $90^o$ in steps of $5^o$. The laminate was considered symmetric and balanced. Being symmetric is a practical assumption which is a great advantage in problem simplification as only half of the laminate is needed for optimization. In addition, the requirement that the laminate is balanced can be easily enforced by using pairs of ±θ layers at symmetric state.

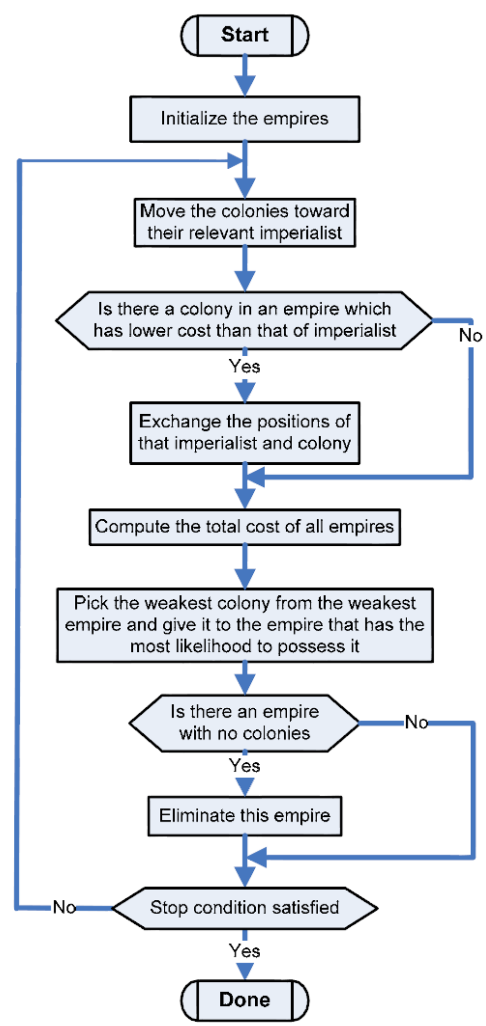



**Figure 3.** Flowchart of the proposed algorithm [26]

**Table 1.** The numbers for glass/epoxy and carbon/epoxy materials with related angles.

| Angles | Glass/epoxy | Carbon/epoxy |
|---|---|---|
| 0 | 1 | 20 |
| +5/-5 | 2 | 21 |
| +10/-10 | 3 | 22 |
| +15/-15 | 4 | 23 |
| +20/-20 | 5 | 24 |
| +25/-25 | 6 | 25 |
| +30/-30 | 7 | 26 |
| +35/-35 | 8 | 27 |
| +40/-40 | 9 | 28 |
| +45/-45 | 10 | 29 |
| +50/-50 | 11 | 30 |
| +55/-55 | 12 | 31 |
| +60/-60 | 13 | 32 |
| +65/-65 | 14 | 33 |
| +70/-70 | 15 | 34 |
| +75/-75 | 16 | 35 |
| +80/-80 | 17 | 36 |
| +85/-85 | 18 | 37 |
| 90 | 19 | 38 |

This assumption was taken in order to minimize shear-extension and bending-twisting effects. Although $0^\circ$ plies and $90^\circ$ layers may not need to come in pairs, they are treated like other angles due to programming necessities but with half the normal thickness to simulate a single ply. The numbers of pair layers vary from 6 to 11 for half of the Laminate.

Table 1 shows the numbers designated to each material and related angles. As present in Table 1, numbers 0 to 19 and 20 to 38 indicate the glass/epoxy and the carbon/epoxy materials, respectively, with corresponding angles.

Multi-objective optimization is an important research topic for researchers. This is due to the multi-objective nature of real world problems. It is difficult to compare the results of one multi-objective method to another, because there is not a unique optimum in multi-objective optimization as in single objective optimization. Hence, the best solution in multi-objective terms may be decided by the decision makers. Recently, multi-objective metaheuristic and evolutionary procedures have become very popular for multi-objective optimization.

The increasing acceptance of metaheuristic algorithms is due to their ability to: (1) find multiple solutions in a single run, (2) work without derivatives, (3) converge speedily to Pareto-optimal solutions with a high degree of accuracy, (4) handle both continuous function and combinatorial optimization problems with ease, (5) be less susceptible to the shape or continuity of the Pareto trade-off curve. These issues are a real concern for the techniques of mathematical programming [45].

In this paper, the Pareto set is generated by optimizing a convex combination of the two objectives, weight ($W$) and cost ($C$) for a series of values for the multiplier $\alpha$ as:

$$F = \alpha W + (1-\alpha)C \tag{20}$$

$$W = abt\,(N_{Ca} \times \rho_{Ca} + N_{Gl} \times \rho_{Gl}) \tag{21}$$

$$C = abt\,(8N_{Ca} \times \rho_{Ca} + N_{Gl} \times \rho_{Gl}) \tag{22}$$

where $a$, $b$, $t$, $N_{Ca}$, $N_{Gl}$, $\rho_{Ca}$, and $\rho_{Gl}$ are the length, width, thickness, the number of carbon/epoxy layer, the number of glass/epoxy layer, the density of carbon/epoxy and glass/epoxy materials, respectively.

## 5. Numerical Results

The properties of glass/epoxy and carbon/epoxy laminates are presented in Table 2 [46]. A code with a system of 100 countries and 3 imperialists was developed in MATLAB 2011 based on the ICA. The performance of ICA is satisfactory using the revolution rate of 0.4 and ξ=0.02. Each stacking sequence is treated as one country. The objective function (F) of any country is calculated based on laminate stacking sequence (angle, material and situation of each layer) and ICA process finally lead to best stacking sequence in 500 iterations.

According to the literature [30, 31], by assigning $\alpha$ = 0, 0.7, 0.8, 0.87, 0.93, 0.96, and 1, the combined objective function (F) is minimized by using a single-objective optimizer based on ICA. Running the program for 10 times in order to ensure the convergence, the best results were obtained and compared with GA and ACS. The optimal stacking sequences and optimum values of the multi-objective function for $\alpha$ = 0 are given in Table 3. Figure 4 shows the value of multi-objective function ($F$) with respect to the number of iterations for $\alpha$ = 0. Pair 11 with 42 layers was obtained as an optimal number of layers for multi-objective function with $\alpha$ = 0.

By observing Fig. 4, the convergence was achieved in less than 110 iterations. The optimal stacking sequence and optimum values of multi-objective function for $\alpha$ = 0.7, 0.8, 0.87, 0.93 and 0.96 are given in Table 4.

**Table 2.** Mechanical properties of glass/epoxy and carbon/epoxy.

| Parameters | Carbon /epoxy | Glass /epoxy |
|---|---|---|
| Longitudinal modulus (GPa) | 137.9 | 43.4 |
| Transverse modulus (GPa) | 8.96 | 8.89 |
| In-plane shear modulus (GPa) | 7.1 | 4.55 |
| Poisson ratio | 0.3 | 0.27 |
| Material density (kg/m$^3$) | 1587 | 1970 |
| Layer thickness (mm) | 0.127 | 0.127 |
| Cost factor | 8 | 1 |

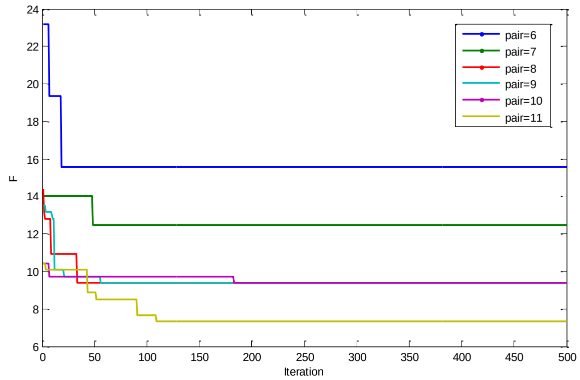



**Figure 4.** Multi-objective function value (F) with respect to the number of iterations for $\alpha = 0$

Figure 5 shows the value of multi-objective function with respect to the number of iterations for $\alpha$ = 0.7, 0.8, 0.87, 0.93, 0.96. Pairs 8 and 9 with 32 layers were obtained as an optimal number of layers for multi-objective function with $\alpha$ = 0.7. Pairs 6, 7 and 8 with 24 layers were obtained as an optimal number of layers for $F$ with $\alpha$ = 0.8 and $\alpha$ = 0.87. Pairs 6, 7 and 8 with 22 layers were obtained as an optimal number of layers for $F$ with $\alpha$ = 0.93. Pairs 6, 7, 8 and 9 with 22 layers were obtained for $\alpha$ = 0.96.

The optimal stacking sequence and optimum value of $F$ for $\alpha$ = 1 is given in Table 5. The values of multi-objective function versus the number of iterations for $\alpha$ = 1 are shown in Fig. 5. Pairs 6, 7, 8, 9 and 10 with 22 layers were obtained as an optimal number of layers for optimum multi-objective function with $\alpha$ = 1. As can be seen, the convergence was achieved in less than 50 iterations.

## 6. Discussion

The number of layers and materials are two important factors for multi-objective optimization. Also, the angles of composite layers play a role in determining the first natural frequency. When the value of weighting factor ($\alpha$) is set to zero, the problem is reduced to single-objective optimizing problem for cost minimization. The ICA can be used for a laminate with the layers which is completely made of glass/epoxy plies as is shown in Table 3. For $\alpha$ = 1, the only active objective is the weight and consequently a laminate is completely made of carbon/epoxy layers as is shown in Table 5. The carbon/epoxy is stiffer than glass/epoxy, and can fulfil the requirement for the minimum value of the first natural frequency with lesser number of plies.

All applied methods including GA, ACS, and ICA achieve optimum designs in which the layers are made of carbon/epoxy in the outer layers and those are made of glass/epoxy in the inner ones. This creates a sandwich-type composite where the structural function is assured by the stiff carbon layers, placed on the outside, where their contribution to the flexural properties of the laminate is maximal, while inner layers are merely used to increase the distance of the outer plies from the neutral plane and to reduce the total cost.

The Contribution of layers with angles ranging from $\pm 40^\circ$ to $\pm 60^\circ$ is to maximize the first natural frequency of the plate. The appearance of $0^\circ$ or $90^\circ$ plies is due to the reduction in weight and cost. Although these plies may not contribute much to the frequency, it is advantageous to use them. Unlike other angles, they do not come in pairs which save unnecessary additional weight and cost. In addition, the $0^\circ$ plies always come into view in the inner layers where they are the least damaging for the performance of the plate.

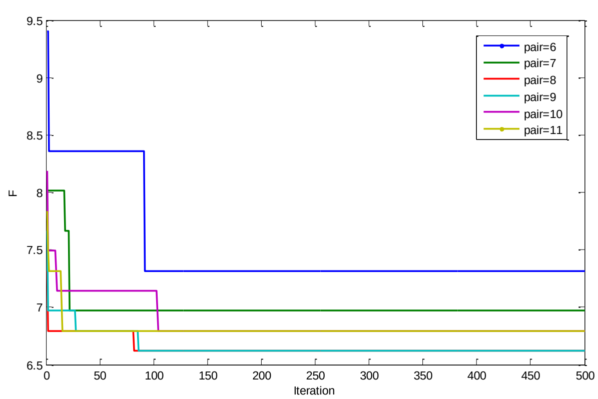
Table 6 shows the comparisons of the ICA, GA, and ACS for each given $\alpha$. It should be noted that with the intention of illustrating the material of each layer in the final stacking sequence notation, the glass/epoxy layers are shown by plain numbers, while the carbon/epoxy layers are represented by underlined numbers.

**Table 3.** Stacking sequences and optimum values for multi-objective function with $\alpha = 0$.

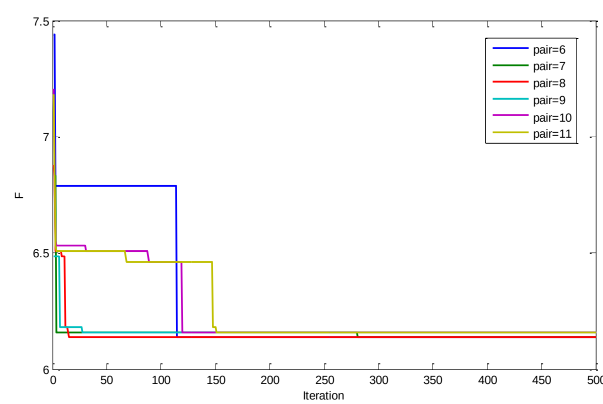
| Best | Stacking sequences | Cost | Weight (Kg) | Frequency (Hz) | $F$ |
|---|---|---|---|---|---|
| 1 | 10 4 15 16 11 9 15 4 18 11 1 | 7.3217 | 7.3217 | 25.0842 | 7.3217 |
| 2 | 4 14 7 4 7 9 15 5 1 11 11 | | | 25.2398 | |
| 3 | 14 10 15 18 14 4 5 3 3 4 1 | | | 25.2871 | |
| 4 | 9 2 7 15 15 12 5 3 1 11 7 | | | 25.1706 | |
| 5 | 9 2 7 15 15 12 5 3 1 11 7 | | | 25.0668 | |
| 6 | 14 4 10 9 13 2 19 2 16 5 14 | | | 25.3650 | |
| 7 | 9 3 10 12 5 8 10 16 2 1 10 | | | 25.0762 | |
| 8 | 10 14 19 10 13 5 3 4 17 13 4 | | | 25.2852 | |
| 9 | 4 3 4 4 10 17 13 6 1 3 8 | | | 25.0613 | |
| 10 | 10 10 4 2 5 3 9 17 17 9 1 | | | 25.4732 | |

**Table 4.** Stacking sequences and optimum values for multi-objective function for α = 0.7, 0.8, 0.87, 0.93 and 0.96

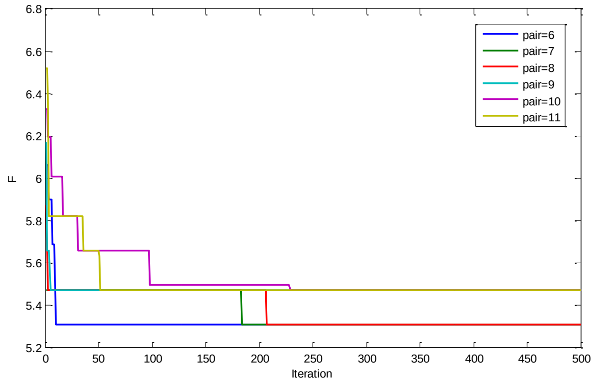
| $\alpha$ | Best | Stacking sequences | Cost | Weight (Kg) | Frequency (Hz) | $F$ |
|---|---|---|---|---|---|---|
| | 1 | 34 9 5 7 12 12 9 9 | | | 25.0256 | |
| | 2 | 28 9 9 14 4 16 10 6 | | | 25.3566 | |
| | 3 | 33 15 3 4 14 8 13 4 | | | 25.1308 | |
| 0.7 | 4 | 28 9 10 15 7 5 3 5 | 9.3750 | 5.4429 | 25.2970 | 6.6225 |
| | 5 | 33 10 14 15 17 5 1 9 1 | | | 25.1993 | |
| | 6 | 28 4 10 2 3 1 7 14 1 | | | 25.0591 | |
| | 7 | 28 4 3 19 10 14 1 5 15 | | | 25.1328 | |
| | 8 | 28 9 17 4 14 17 1 4 1 | | | 25.0520 | |
| | 1 | 33 28 33 4 11 14 | | | 25.0982 | |
| | 2 | 28 28 34 17 18 7 | | | 25.0745 | |
| | 3 | 28 33 28 14 3 17 | | | 25.1695 | |
| | 4 | 28 34 33 18 17 4 | | | 25.0133 | |
| 0.8 | 5 | 28 29 23 10 2 5 | 15.5736 | 3.7771 | 25.0089 | 6.1364 |
| | 6 | 29 28 34 9 13 17 | | | 25.0659 | |
| | 7 | 28 34 23 3 5 8 | | | 25.0624 | |
| | 8 | 28 34 38 38 7 14 12 | | | 25.0183 | |
| | 9 | 28 28 38 38 9 16 17 | | | 25.0481 | |
| | 10 | 34 28 34 15 1 1 1 1 | | | 25.1675 | |
| | 1 | 28 33 34 14 14 10 | | | 25.1975 | |
| | 2 | 28 29 34 15 7 5 | | | 25.1254 | |
| | 3 | 28 28 34 15 14 15 | | | 25.2269 | |
| | 4 | 34 28 28 16 5 8 | | | 25.1167 | |
| | 5 | 29 28 29 5 14 14 | 15.5736 | 3.7771 | 25.0360 | 5.3107 |
| 0.87 | 6 | 34 28 28 1 11 2 19 | | | 25.0177 | |
| | 7 | 28 38 34 38 15 2 3 | | | 25.0350 | |
| | 8 | 33 34 38 38 3 3 14 | | | 25.0035 | |
| | 9 | 34 34 38 38 14 5 16 | | | 25.1040 | |
| | 10 | 34 38 34 38 10 19 1 15 | | | 25.0397 | |
| | 1 | 34 29 23 28 23 19 | | | 25.0825 | |
| | 2 | 28 28 34 31 24 19 | | | 25.0074 | |
| | 3 | 34 28 29 23 1 35 | | | 25.0132 | |
| | 4 | 28 34 24 28 24 1 | | | 25.0705 | |
| 0.93 | 5 | 34 28 34 24 27 1 | 22.8181 | 3.1573 | 25.1259 | 4.5336 |
| | 6 | 34 28 34 34 23 1 | | | 25.2532 | |
| | 7 | 34 34 29 28 1 21 | | | 25.0803 | |
| | 8 | 34 28 29 38 26 20 1 | | | 25.0338 | |
| | 9 | 28 28 24 38 34 38 1 | | | 25.0434 | |
| | 10 | 28 33 23 38 38 38 38 1 | | | 25.0536 | |
| | 1 | 28 33 29 28 23 1 | | | 25.1622 | |
| | 2 | 28 34 33 34 1 22 | | | 25.0898 | |
| | 3 | 28 33 28 21 35 19 | | | 25.0006 | |
| | 4 | 34 29 23 33 23 19 | | | 25.0767 | |
| 0.96 | 5 | 34 33 23 33 27 1 | 22.8181 | 3.1573 | 25.0668 | 3.9438 |
| | 6 | 34 34 28 34 1 33 | | | 25.1325 | |
| | 7 | 28 34 33 38 38 1 35 | | | 25.0232 | |
| | 8 | 28 38 34 38 28 31 19 | | | 25.0291 | |
| | 9 | 33 34 38 38 28 38 38 19 | | | 25.0867 | |
| | 10 | 34 33 38 38 38 38 38 1 38 | | | 25.0119 | |



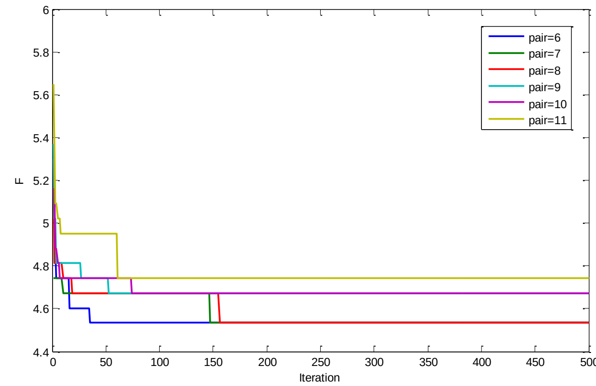

$\alpha = 0.7$



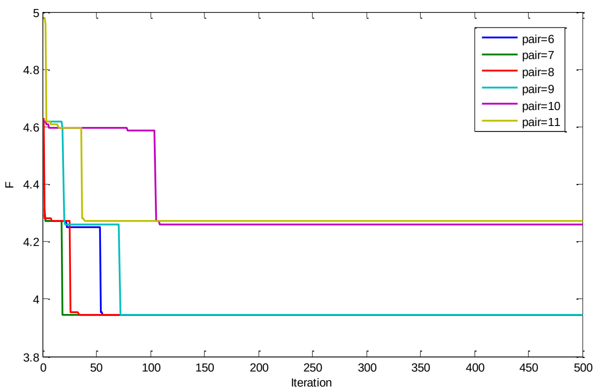

$\alpha = 0.8$



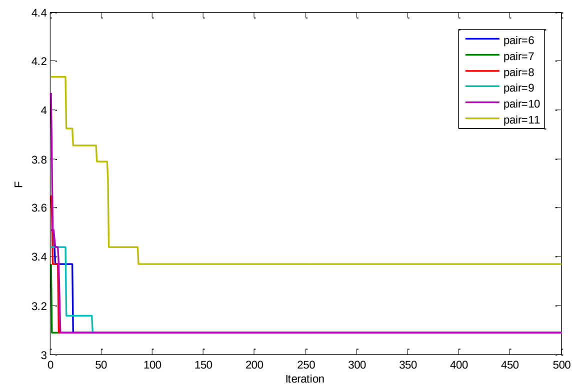

$\alpha = 0.87$




$\alpha = 0.93$




$\alpha = 0.96$




$\alpha = 1$

**Figure 5.** Multi-objective function value (F) with respect to the number of iterations for $\alpha = 0.7, 0.8, 0.87, 0.93, 0.96$ and 1

**Table 5**. Stacking sequences and optimum values for F with α = 1.

| Best | Stacking sequences | Cost | Weight (Kg) | Frequency (Hz) | *F* |
|---|---|---|---|---|---|
| 1 | 28 23 23 29 29 38 | | | 25.2313 | |
| 2 | 23 33 38 29 34 33 | | | 25.1387 | |
| 3 | 34 28 38 23 32 22 | | | 25.1849 | |
| 4 | 23 33 38 34 26 28 | | | 25.0577 | |
| 5 | 38 33 29 28 27 29 | | | 25.1193 | |
| 6 | 28 33 29 38 34 22 | | | 25.4151 | |
| 7 | 34 28 28 38 38 38 28 | | | 25.4765 | |
| 8 | 38 28 38 28 26 21 38 | | | 25.0230 | |
| 9 | 29 38 29 38 22 24 38 | | | 25.0450 | |
| 10 | 38 38 28 28 29 28 20 | 24.7164 | 3.0895 | 25.1800 | 3.0895 |
| 11 | 38 29 28 38 38 38 38 32 | | | 25.1183 | |
| 12 | 28 38 38 38 38 38 24 33 | | | 25.1277 | |
| 13 | 38 28 23 38 38 38 38 23 | | | 25.1021 | |
| 14 | 38 33 38 38 38 38 38 38 25 | | | 25.0101 | |
| 15 | 29 38 38 38 38 38 38 38 35 | | | 25.0079 | |
| 16 | 38 34 38 38 38 38 38 38 22 | | | 25.0854 | |
| 17 | 38 38 28 38 38 38 38 38 38 38 | | | 25.0217 | |
| 18 | 34 38 38 38 38 38 38 38 20 38 | | | 25.1227 | |
| 19 | 28 38 38 38 38 38 38 38 38 38 | | | 25.1359 | |
| 20 | 38 38 34 38 38 38 38 38 38 38 | | | 25.0346 | |

**Table 6.** Comparisons of optimization results for the ICA against GA [30] and ACS [31]

| $\alpha$ | Method | Pairs number of half of Laminate | Stacking sequences | Cost | Weight (Kg) | Frequency (Hz) | $F$ |
|---|---|---|---|---|---|---|---|
| 0 | ICA | 11 | $[\pm45_2/\pm15/\pm5/\pm20/\pm10/\pm40/\pm80_2/\pm40/0]_S$ | 7.32 | 7.32 | 25.47 | 7.32 |
| | GA | 11 | $[\pm50_{10}/0]_S$ | 7.32 | 7.32 | 25.82 | 7.32 |
| | ACS | 11 | $[\pm55/\pm50/\pm65/90/\pm25/\pm85_2/\pm75/\pm85/\pm60/\pm50]_S$ | 7.32 | 7.32 | 25.07 | 7.32 |
| 0.7 | ICA | 8 | $[\pm\underline{40}/\pm40/\pm45/\pm70/\pm30/\pm20/\pm10/\pm20]_S$ | 9.37 | 5.44 | 25.29 | 6.62 |
| | | 9 | $[\pm\underline{65}/\pm45/\pm65/\pm70/\pm80/\pm20/0/\pm40/0]_S$ | 9.37 | 5.44 | 25.19 | 6.62 |
| | GA | 8 | $[\pm\underline{50}/\pm50_7]_S$ | 9.37 | 5.44 | 25.10 | 6.62 |
| | ACS | 8 | $[\pm\underline{50}/\pm50_2/\pm30/\pm65/\pm40/\pm70/\pm40]_S$ | 9.37 | 5.44 | 25.09 | 6.62 |
| 0.8 | ICA | 6 | $[\pm\underline{40}/\pm\underline{65}/\pm\underline{40}/\pm65/\pm10/\pm80]_S$ | 15.57 | 3.77 | 25.16 | 6.13 |
| | | 7 | $[\pm\underline{40}_2/\underline{90}_2/\pm40/\pm75/\pm80]_S$ | 15.57 | 3.77 | 25.04 | 6.13 |
| | | 8 | $[\pm\underline{70}/\pm\underline{40}/\pm\underline{70}/\pm70/0_4]_S$ | 15.57 | 3.77 | 25.16 | 6.13 |
| | GA | 7 | $[\pm\underline{50}_2/\pm50_5]_S$ | 12.52 | 4.61 | 25.88 | 6.19 |
| | ACS | 7 | $[\pm\underline{60}/\pm\underline{40}/\pm45/\pm85/\pm65/\pm85/\pm5]_S$ | 12.52 | 4.61 | 25.42 | 6.19 |
| 0.87 | ICA | 6 | $[\pm\underline{40}_2/\pm\underline{70}/\pm70/\pm65/\pm70]_S$ | 15.57 | 3.77 | 25.22 | 5.31 |
| | | 7 | $[\pm\underline{40}/\underline{90}/\pm\underline{70}/\underline{90}/\pm70/\pm5/\pm10]_S$ | 15.57 | 3.77 | 25.03 | 5.31 |
| | | 8 | $[\pm\underline{70}/\underline{90}/\pm\underline{70}/\underline{90}/\pm45/90/0/\pm70]_S$ | 15.57 | 3.77 | 25.03 | 5.31 |
| | GA | 7 | $[\pm\underline{45}_2/\underline{90}/\pm50_3/\pm80]_S$ | 14.02 | 4.19 | 25.08 | 5.47 |
| | ACS | 7 | $[\pm\underline{50}/\pm\underline{40}/\underline{90}/\pm50/\pm65/\pm75/\pm45]_S$ | 14.02 | 4.19 | 25.11 | 5.47 |
| 0.93 | ICA | 6 | $[\pm\underline{70}/\pm\underline{40}/\pm\underline{70}_2/\pm\underline{15}/0]_S$ | 22.81 | 3.15 | 25.25 | 4.53 |
| | | 7 | $[\pm\underline{70}/\pm\underline{40}/\pm\underline{45}/\underline{90}/\pm\underline{30}/\underline{0}/0]_S$ | 22.81 | 3.15 | 25.03 | 4.53 |
| | | 8 | $[\pm\underline{40}/\pm\underline{65}/\pm\underline{15}/\underline{90}_4/0]_S$ | 22.81 | 3.15 | 25.05 | 4.53 |
| | GA | 7 | $[\pm\underline{50}_3/\underline{90}/\pm50_2/0]_S$ | 17.47 | 3.71 | 25.38 | 4.67 |
| | ACS | 6 | $[\pm\underline{55}_2/\pm\underline{40}_2/\underline{90}/\pm\underline{65}]_S$ | 24.72 | 3.09 | 25.02 | 4.60 |
| 0 .96 | ICA | 6 | $[\pm\underline{40}/\pm\underline{65}/\pm\underline{45}/\pm\underline{40}/\pm\underline{15}/0]_S$ | 22.81 | 3.15 | 25.16 | 3.94 |
| | | 7 | $[\pm\underline{40}/\underline{90}/\pm\underline{70}/\underline{90}/\pm\underline{40}/\pm\underline{55}/90]_S$ | 22.81 | 3.15 | 25.02 | 3.94 |
| | | 8 | $[\pm\underline{65}/\pm\underline{70}/\underline{90}_2/\pm\underline{40}/\underline{90}_2/90]_S$ | 22.81 | 3.15 | 25.08 | 3.94 |
| | | 9 | $[\pm\underline{70}/\pm\underline{65}/\underline{90}_5/0/\underline{90}]_S$ | 22.81 | 3.15 | 25.01 | 3.94 |
| | GA | 6 | $[\pm\underline{50}_4/\pm50_2]_S$ | 19.37 | 3.64 | 26.07 | 4.27 |
| | ACS | 6 | $[\pm\underline{55}/\pm\underline{50}/\pm\underline{55}/\pm\underline{60}/\underline{0}/\pm\underline{5}]_S$ | 24.72 | 3.09 | 27.07 | 3.95 |
| 1 | ICA | 6 | $[\pm\underline{40}/\pm\underline{65}/\pm\underline{45}/\underline{90}/\pm\underline{70}/\pm\underline{10}]_S$ | 24.71 | 3.08 | 25.41 | 3.08 |
| | | 7 | $[\pm\underline{70}/\pm\underline{40}_2/\underline{90}_3/\pm\underline{40}]_S$ | 24.71 | 3.08 | 25.47 | 3.08 |
| | | 8 | $[\pm\underline{40}/\underline{90}_5/\pm\underline{20}/\pm\underline{65}]_S$ | 24.71 | 3.08 | 25.12 | 3.08 |
| | | 9 | $[\underline{90}/\pm\underline{70}/\underline{90}_6/\pm\underline{10}]_S$ | 24.71 | 3.08 | 25.08 | 3.08 |
| | | 10 | $[\pm\underline{70}/\underline{90}_7/\underline{0}/\underline{90}]_S$ | 24.71 | 3.08 | 25.12 | 3.08 |
| | GA | 6 | $[\pm\underline{50}_5/\underline{0}]_S$ | 24.72 | 3.09 | 25.14 | 3.09 |
| | ACS | 6 | $[\pm\underline{55}/\pm\underline{50}/\pm\underline{55}/\underline{90}/\pm\underline{55}/\pm\underline{35}]_S$ | 24.72 | 3.09 | 25.10 | 3.09 |

The quantities in the GA were converted to the SI system. It can be seen that the ICA has outperformed the GA and ASC. Furthermore, in terms of the constraint accuracy, the ICA is superior over GA and ACS algorithms. However, the main advantage of the GA over both ACS and ICA is the orientation of angles which makes them easier to construct laminated composites.

## 7. Conclusions

The problem of obtaining minimum cost and weight using ICA in hybrid laminate composites was investigated. The symmetric balanced laminate made of glass/epoxy or/and carbon/epoxy layers was chosen with certain geometrical specifications. The material and number of layers were the design of variables as well as the fiber orientations. The optimizing process was constrained by the first natural frequency of the plate to be not less than a predefined value (25 Hz). The results were evaluated for different weighting factors ($\alpha$) based on WSM and compared to those obtained by the GA and ACS. This comparison confirms the superiority of the ICA in terms of the function's value and constraint accuracy over GA and ACO. When cost was a primary consideration the plate was made from glass/epoxy, and when weight was a primary consideration, it was made of carbon/epoxy. The less stiff glass/epoxy layers, when they have to be used, always appear in the inner layers. This creates a sandwich type composite where the structural function is assured by the stiff carbon layers, placed on the outside, where their contribution to the flexural properties of the laminate is maximal/maximized, while inner layers are merely used to increase the distance of the outer plies from the medial plane. It certainly cannot be stated that metaheuristic algorithms have advantages over each other, since it depends on the structure of the problems and algorithm parameters. The ICA has reached to the same sequences and in some cases has reached to the better sequences of mentioned algorithms and it has offered minimum values for multi-objective function. This algorithm is so useful and as a new method is competitive with other heuristic algorithms.

## References


[1] Ghasemi H, Roberto Brighenti R, Xiaoying Zhuang X, Jacob Muthu J, Timon Rabczuk T. Optimization of fiber distribution in fiber reinforced composite by using NURBS functions. Comput Mater Sci 2014; 83: 463-473.

[2] Blasques J P, Stolpe M. Maximum stiffness and minimum weight optimization of laminated composite beams using continuous fiber angles. Struct Multidiscip O 2011; 43(4): 573-588.

[3] Sadollah A, Bahreininejad A, Hamdi M, Purbolaksono J. Optimum mechanical behavior of calcium phosphate cement/hydroxyl group functionalized multi-walled carbon nanotubes/bovine serum albumin composite using metaheuristic algorithms. Neural Comput Appl 2014; 24(1): 193-200.

[4] Zuo Z H, Xie Y M. Maximizing the effective stiffness of laminate composite materials. Comput Mater Sci 2014; 83: 57-63.

[5] Hemmatian H, Fereidoon A, Sadollah A, Bahreininejad A. Optimization of laminate stacking sequence for minimizing weight and cost using elitist ant system optimization. Adv Eng Softw 2013; 57: 8-18.

[6] Sobey AJ, Blake JIR, Shenoi RA. Implications of failure criteria choices on the rapid concept design of composite grillage structures using multiobjective optimisation. Struct Multidiscip O 2013; 47: 735-747.

[7] Omkar SN, Venkatesh A, Mudigere M. MPI-based parallel synchronous vector evaluated particle swarm optimization for multi-objective design optimization of composite structures. Eng Appl Artif Intel 2012; 25: 1611-1627.

[8] Rahul, S G, Chakraborty D, Dutta A. Multi-objective optimization of hybrid laminates subjected to transverse impact. Compos Struct 2006; 73(3): 360-369.

[9] Javidrad F, Nouri R. A simulated annealing method for design of laminates with required stiffness properties. Compos Struct 2011; 93: 1127-35.

[10] Gubran HBH, Gupta K. The effect of stacking sequence and coupling mechanisms on the natural frequencies of composite shafts. Sound Vib 2012; 282: 231-248.

[11] Banerjee JR. Explicit analytical expressions for frequency equation and mode shapes of composite beams. Sol Struct 2001; 38: 2415-2426.

[12] Shun F H, Chao SC. Determination of elastic constants of materials by vibration testing. Compos Struct 2000; 49(2): 183-190.

[13] Abachizadeh M, Tahani M. An ant colony optimization approach to multi objective optimal design of symmetric hybrid laminates for maximum fundamental frequency and minimum cost. Struct Multidisc Optim 2009; 37: 367-376.

[14] Abdalla MM, Setoodeh S, Gürdal Z. Design of variable stiffness composite panels for maximum fundamental frequency using lamination parameters. Compos Struct 2007; 81: 283-291.

[15] Bert CW. Optimal design of a composite-material plate to maximize its fundamental frequency. Sound Vib 1977; 50: 229-237.

[16] Reiss R, Ramachandran S. Maximum frequency design of symmetric angle-ply laminates. Compos Struct 1987; 4: 1476-1487.

[17] Grenestedt JL. Layup optimization and sensitivity analysis of the fundamental eigenfrequency of composite plates. Compos Struct 1989. 12: 193-209.

[18] Duffy KJ, Adali S. Maximum frequency design of pre-stressed symmetric, cross-ply laminates of hybrid construction. Adv Des Auto 1991; 2: 477-484.

[19] Adali S. Design of shear-deformable antisymmetric angle-ply laminates to maximize the fundamental frequency and frequency separation. Compos Struct 1984; 2: 349-369.

[20] Apalak MK, Yildirim M, Ekici R. Layer optimization for maximum fundamental frequency of laminated composite plates for different edge conditions. Compos Sci Technol 2008; 68: 537-550.

[21] Fukunaga H, Sekine H, Sato M. Optimal design of symmetric laminated plates for fundamental frequency. Sound Vib 1994; 171: 219-229.

[22] Narita Y. Layerwise optimization for the maximum fundamental frequency of laminated composite plates. Sound Vib 2003; 263: 1005-1016.

[23] Narita Y, Hodgkinson JM. Layerwise optimisation for maximizing the fundamental frequencies of point-supported rectangular laminated composite plates. Compos Struct 2005; 69: 127-135.

[24] Adali S, Duffy KJ. Minimum cost design of vibrating laminates by hybridization. Eng Optim 1992; 19: 255-267.

[25] Farshi B, Rabiei R. Optimum design of composite laminates for frequency constraints. Compos Struct 2007; 81(4): 87-597.

[26] Adali S, Verijenko VE. Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations. Compos Struct 2001; 54: 131-138.

[27] Tahani M, Kolahan F, Sarhadi A. Genetic algorithm for multi-objective optimal design of sandwich composite laminates with minimum cost and maximum frequency. ICMPM, Sathyamangalam, India, 2005. p. 741-748.

[28] Kolahan F, Tahani M, Sarhadi A. Optimal design of sandwich composite laminates for minimum cost and maximum frequency using simulated annealing. TICME, Tehran, Iran, 2005.

[29] Hao W, Ying Y, Yujia L. Reliability Based Optimization of Composite Laminates for Frequency Constraint. Chinese Journal of Aeronautics 2008; 21: 320-327.

[30] Grosset L, Venkataraman S, Haftka RT. Genetic optimization of two-material composite laminates. In: 16th ASC Technical Meeting, Blacksburg, Virginia, 2001.

[31] Abachizadeh M, Tahani M. Ant colony optimization of hybrid laminates for minimum cost and weight. In: Saravanos DA (ed) Proceedings of 6th international symposium on advanced composites COMP’07, Corfu, Greece, 2007.

[32] Atashpaz-Gargari E, Lucas C. Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition. In: IEEE Congress on Evolutionary Computation, Singapore, 2007. p. 4661-67.

[33] Atashpaz-Gargari E, Hashemzadeh F, Rajabioun R, Lucas C. Colonial competitive algorithm: A novel approach for PID controller design in MIMO distillation column process. Int J Intel Comput Cyber 2008; 3: 337-55.

[34] Kaveh A, Talatahari S. Imperialist competitive algorithm for engineering design problems. Asian J Civil Eng 2010; 11: 675-697.

[35] Kaveh A, Talatahari S. Optimum design of skeletal structures using imperialist competitive algorithm. Comput Struct 2010; 88: 1220-1229.

[36] Abdi B, Mozafari H, Ayob A, Kohandel R. Imperialist Competitive Algorithm and its Application in Optimization of Laminated Composite Structures. European J Sci Res 2011; 55: 174-187.

[37] Mozafari H, Abdi B, Ayob A. Optimization of Composite Plates Based on Imperialist Competitive Algorithm. Int J Comput Sci Eng 2010; 2: 2816-2819.

[38] Mozafari H, Alias A, Kamali F. Optimum Design of Composite Plates under Thermal Buckling Loads using Imperialist Competitive Algorithm. Int J Comput Sci Eng Technol 2010; 1: 54-58.

[39] Esmaeilzadeh M. A modified colonial competitive algorithm for optimizing convex functions. *Int J Intel Comput Cyber* 2013; 6(4): 370-385.

[40] Reddy JN. Mechanics of laminated composite plates and shells: Theory and analysis. 2nd ed. CRC Press Inc., Boca Raton, Florida, 2004.

[41] Biabangard-Oskouyi A, Atashpaz-Gargari E, Soltani N, Lucas C. Application of Imperialist Competitive Algorithm for materials property characterization from sharp indentation test. Int J Eng Simul 2008; 10.

[42] Sepehri Rad H, Lucas C. Application of Imperialistic Competition Algorithm in Recommender Systems. In: 13th Int'l CSI Computer Conference (CSICC'08), Kish Island, Iran, 2008.

[43] Rajabioun R, Atashpaz-Gargari E, Lucas C. Colonial Competitive Algorithm as a Tool for Nash Equilibrium Point Achievement. Lect notes comput sc 2008; 5073: 680-695.

[44] Niknam T, Taherian Fard E, Pourjafarian N, Rousta A. An efficient hybrid algorithm based on modified imperialist competitive algorithm and K-means for data clustering. Eng Appl Artif Intel 2011; 24: 306-317.

[45] Suman B, Kumar P. A survey of simulated annealing as a tool for single and multiobjective optimization. Oper Res Soc 2006; 15: 1143-1160.

[46] Herakovich CT. Mechanics of fiberous composites, New York: John Whiley and Sons, 1998