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Propagation of Matrix Cracking and Induced Delaminatin in Cross-Ply Composite Beams Subjected to Bending Loads

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ABSTRACT

Due to the mismatch of mechanical properties in composite laminates, propagation of delamination is considered as a severe damage mechanism in beams with various lay-up configurations. Delamination can be generated due to matrix cracking propagation or it can also be initiated due to the manufacturing process before using composite beams. Using a micromechanics model, this study is aimed to investigate the bending moment required for matrix cracking and induced delamination in cross-ply composite beams. To that end, a unit cell is selected from the lamina surface in a composite beam containing matrix cracking and delamination. Later, the governing equation of stress and displacement fields are extracted in the unit cell to calculate the strain energy release rate due to the propagation of matrix cracking and induced delamination. In order to validate this method, the stiffness variations in Carbon-Epoxy cross-ply laminate $[90/0_2]s$ is examined and the obtained results are compared with the numerical results. It concluded that there is a favorable agreement between the results of the proposed micromechanics model and available numerical results.

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1. Introduction

Matrix cracking and induced delamination are common damage mechanisms in composite laminates yielding stiffness reduction and abrupt failure [1]. Matrix cracking is one of the causes of delamination in laminates. Interlaminar stresses lead to the local delamination propagating through the contact surfaces close to the matrix cracking. Composite laminates are weak in the thickness direction and are sensitive to the loads that result in an increase in the stresses through the thickness. Stresses in the thickness direction can be propagated and expanded on the laminate edges even once there is no external load in the thickness direction [2, 3]. Although these stresses are typically lower than the applied in-plane stresses, they acted in a weak direction and thus would result in the delamination phenomenon [4]. The propagation of the delamination alters the stress distribution within the laminas, affecting the strength, stiffness, and fatigue life. This has provoked many researchers to study the creation and propagation of these damage mechanisms for composite laminates.

The analysis of matrix cracking and induced delamination was performed by Nairn and Hu [5] for cross-ply laminates based on the variational method. Armanios et al. [6] applied the shear deformation theory to the sub-lamina method to analyze the local delamination induced by the transverse crack in CFRP. The proposed model in this study predicted the thermal-moisture effects in agreement with the experimental results of general strain attributable to the initiation of delamination in [7]. Funkunaga et al. [8] presented a one-dimensional "shear lag" analysis using the shear intralaminar concept for the laminates containing delamination generated from the

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transverse crack tip. Dharani and Tang [9] took advantage of this method to analyze the stress of a laminate with delamination about a transverse crack. Zhang et al. [10] benefited from the 2D “modified shear lag” method to predict the strain energy release rate for delamination on the edge and tip of a crack in laminates $[\pm\theta m/90n]_s$. Furthermore, for local delamination, the strain energy release rate was assumed as a function of crack density and delamination area. Moreover, employing the first-order shear theory, Zhang et al. [11] studied the delamination among laminas θ/90 in laminates $[\ldots/\theta i/\theta m/90]_s$ under tension in the direction of sub-laminate. They found that the strain energy release rate in local delamination and the stiffness reduction of transverse laminas are dramatically dependent on the lay-up of local laminas in the damaged laminate. Kashtalyan and Soutis [12-14] evaluated the effects of delamination on the stiffness reduction in a cross-ply laminate. They concluded that the reduction in shear stiffness and Poisson’s ratio of the laminate under the delamination is drastically larger than the longitudinal stiffness reduction under the same condition. Furthermore, Kashtalyan and Soutis [15] theoretically investigated the uniform propagation of delamination induced by the transverse crack in the angle-ply laminates subjected to tensile loads. They obtained a closed analytical solution to the strain energy release rate as a function of damage density and damage area. Pupurs et al. [16] analytically investigated the bending stiffness variations, due to the matrix cracking and induced delamination in cross-ply laminates, and also compared the obtained results with the results of FEM model. Zubillaga et al. [17] presented a new failure criterion to evaluate the matrix cracking induced delamination in laminated composites. Zubillaga et al. also introduced an experimental method to investigate the occurrence of matrix cracking and its effect on the delamination [18]. Based on the energy release rate due to crack propagation in cross-ply laminates and the FEM, Curiel-Sosa et al. [19] introduced a method for fracture and delamination simulation. Liu et al. [20] investigated the effects of pre-delamination on the flexural response of $[+45/-45/0]_{2s}$ carbon fiber reinforced polymer (CFRP) laminates. In order to meet the defects of the proposed models, this study aims to develop a micromechanics model capable of analyzing the variations in strain energy release rate and bending moment required for the propagation of transverse cracking and induced delamination at different crack densities in cross-ply composite beams. It should be noted that in this article, for the first time, a closed relation is presented to analyze the propagation of matrix cracking and induced delamination based on the moment applied to the beam. Identical to the research conducted by Farrokhabadi et al. [21-22], a micromechanics model has been applied to calculate the stress and displacements distributions in a single lamina from a general beam containing matrix cracking and induced delamination. Furthermore, the proposed method in this study not only can assess the effects of bending moment on the propagation of matrix cracking and induced delamination based on the changes of the crack density of cross-ply composite beams, but also is able to predict the occurrence of all described damage mechanisms.

2. Methodology

2.1. Geometry and Conditions of the Problem

Matrix cracks are initiated in the cross-ply composite beams under bending load on the tensile side of the beam in 90° laminas. As it can be seen in Fig 1, if a beam containing a damaged lamina is considered under bending, then from which one can extract a unit cell being subjected to the relatively uniform loading.

It can be assumed in this unit cell that the damaged single lamina is divided into two thin sub-laminas free of matrix crack and one thick sub-lamina containing matrix crack. Furthermore, interlaminar shear stresses in both longitudinal and transverse direction, σ^i_{xz} and σ^i_{yz} respectively, are considered at the interface between these two types of sub-laminas in the form of a piece-wise linear distribution as a function of length and thickness of the lamina.

$$\sigma^i_{xz} = f_i(x.z) \qquad \sigma^i_{yz} = g_i(x.z) \tag{1}$$

It is evident that the involved distribution must satisfy the continuity condition in the boundary of sub-laminas. Regarding this distribution and using the equilibrium equations, the stress distributions σ_{xx} , σ_{zz}, and σ_{xy} can be obtained piece-wisely in each sub-lamina based on the functions $f_i(x.z)$ and $g_i(x.z)$.

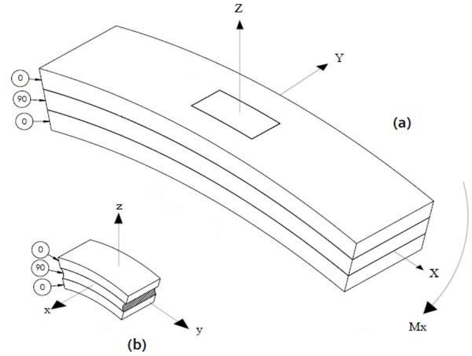

Fig. 1. (a) cross-ply beam under bending load in the reference coordinate system (b) extracted unit cell in the local coordinate system of 90° single lamina.

In what follows, using constitutive law, strain kinematic equation in each sub-lamina, and the assumption made for the shear stresses between sub-laminas, the displacement of different sub-lamina can be determined as a recurrence relation in terms of $f_i(x.z)$ and $g_i(x.z)$. Finally, knowing the stress-strain field of the single lamina, its stiffness reduction would be calculated.

2.2. Matrix Crack and Induce Delamination

The matrix crack and induced delamination is usually triggered by the high values of stress at the contact surface between two undirectional laminas at the crack tip. Fig. 2 depicts the delamination formation with a length of L_d that has been initiated at the top and bottom of the crack tip. It is worth noting that the delamination length in this state is cosidered equal for the top and bottom of the single lamina. After the delamination is formed, the unit cell [0/90/0] is divided into three regions including a region from $(L - L_d)$ to L, a region from $-(L - L_d)$ to $-L$, and the third region is located between these two regions of delamination tip, i.e. from $-(L - L_d)$ to $(L - L_d)$. Due to the formation of free surfaces caused by delamination in the first two regions that are related to 90° lamina, they are considered free of load in which the stress has a zero value. Therefore, these two regions could be omitted from the model and a unit cell can be taken into account in the middle lamina where the equivalent properties can be calculated from the model proposed by Farrokhabadi et al. [21].

2.3. Determination of bending moment in a cross-ply composite beam containing matrix crack and induced delamination

The relation between force-moment and strain-curvature is expressed by the matrix **ABD** in composite beams subjected to bending loading where **D** is the bending stiffness matrix. Here, D_{11} is the longitudinal bending stiffness which is required to be known to obtain the beam longitudinal bending modulus.

$$D_{11} = EI \rightarrow E = \frac{D_{11}}{I} = \frac{12D_{11}}{h^3} \tag{2}$$

Also, the strain energy release rate in the beam containing damaged laminas is explained as follows [23]:

$$G = \frac{M^2 L}{2} \frac{dc_B}{dA} \tag{3}$$

where $c_B = \theta/M$ and dA is the cracked region. Rotation θ is defined in terms of curvature K as:

$$\theta = KL \tag{4}$$

where L denoted the length of lamina. Eq. (3) can be expressed as follows [23]:

$$G(\rho) = \frac{M^2 L}{2} \frac{d\left(\frac{K}{M}\right)}{dA} \tag{5}$$

where $\frac{K}{M} = \frac{1}{E_{flex} I}$

Thus, the strain energy release rate in the presence of matrix crack, whose density has increased from initial state 1 to secondary state 2, is finally obtained as given below [23]:

$$G(\rho) = \frac{\frac{M^2 L}{2}\left[\left(\frac{1}{E(\rho)I}\right)_2 - \left(\frac{1}{E(\rho)I}\right)_1\right]}{t_{90} \times w} \tag{6}$$

where M, L, E, and I stand for the bending moment, length of lamina, flexural modulus, and moment of inertia, respectively. Moreover, t_{90} and w are the thickness and width of 90° lamina, respectively.

$$G(\rho) = \frac{\frac{M^2 L}{2}\left[\left(\frac{1}{E(\rho)I}\right)_2 - \left(\frac{1}{E(\rho)I}\right)_1\right]}{2 \times L' \times t_{90} \times w} \tag{7}$$

where L' is the coefficient of delamination length.

3. Result and Discussion

In this section, the variations of bending stiffness, due to the delamination in laminas with layup configuration $[90/0_2]_s$ and delamination length of 0.3 times the thickness, are evaluated using a micromechanics model. Furthermore, the obtained results are compared to the those obtained by the numerical model. Then, the variations of strain energy release rate in cross-ply composite beams are calculated versus the crack density in the presence of matrix cracking and induced delamination with lengths of 0.1, 0.3, and 0.5 times the thickness. Finally, the bending moment required for the propagation of matrix cracking and induced delamination in laminates $[90/0_2]_s$ and $[90/0]_s$ with delamination lengths of 0.1, 0.3, and 0.5 times the thickness is investigated versus the crack density. The material properties are presented in Table 1.

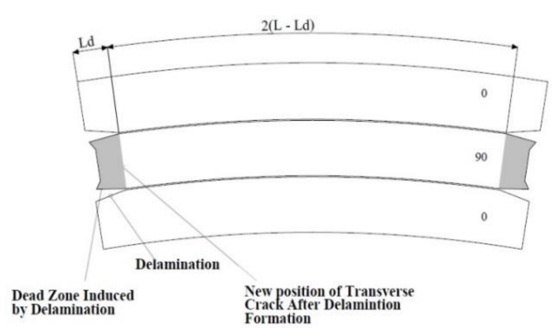

Fig. 2. Unit cell containing matrix crack and delamination.

According to Fig. 3, the bending stiffness is reduced in laminate $[90/0_2]_s$ by an increase in crack density. Furthermore, as it can be observed, an acceptable agreement is achieved between the obtained results from the present study with the results of the numerical model proposed in [16].

3.1. Variations of Strain Energy Release Rate Due to Matrix Cracking and Induced Delamination in Cross-Ply Composite beams

Applying the proposed micromechanics model, Figs. 4 and 5 show the variations of strain energy release rate, due to matrix cracking and induced delamination lengths of 0.1, 0.3, and 0.5 times the thickness, versus crack density in composite laminates $[90/0_2]_s$ and $[90/0]_s$, respectively, under bending moment M=200 N.m. In the present study, length, width, and thickness of each lamina are set to be 12 mm, 7 mm and 0.125 mm respectively. It is worth noting that "td" denotes the thickness of 90° lamina in the represented results.

Table 1. Mechanical properties of the Carbon-Epoxy laminate [16]

E_1 (GPa)	E_2 (GPa)	ν_{12}	G_{12} (GPa)
104	6.14	0.4	5

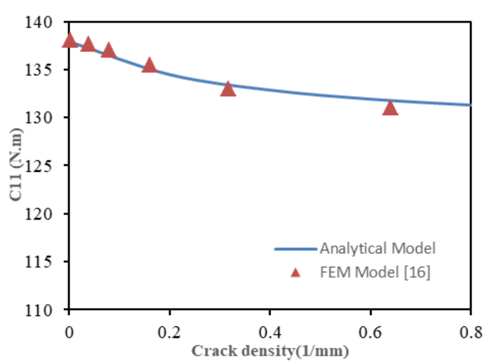

Fig. 3. Comparison of stiffness reduction versus crack density of laminate $[90/0_2]_s$ with a delamination length of 0.3 times the hickness, between the results obtained by the present method and the finite element model.

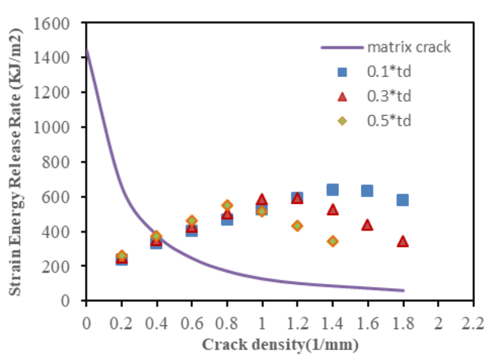

Fig. 4. Variations of strain energy release due to matrix cracking and delamination in composite laminate with stacking sequence $[90/0_2]_s$.

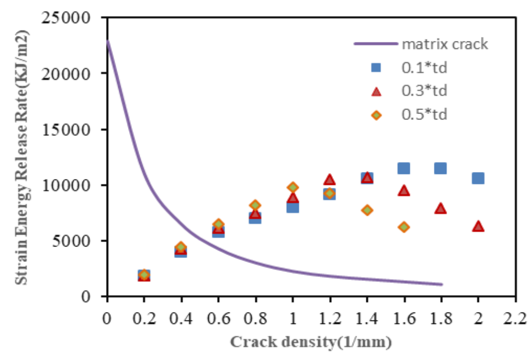

Fig. 5. Variations of strain energy release due to matrix cracking and delamination in composite laminate with stacking sequence $[90/0]_s$.

According to Figs. 4 and 5, as the crack density increases, the strain energy release rate, due to matrix cracking, decreases. Furthermore, the strain energy release rate, due to different delamination lengths, increases at first and begins to decrease at different crack densities. Furthermore, it begins to decrease at a lower crack density for larger delamination lengths which can be attributed to more rapid instability in this state. In the investigated laminates, the strain energy release rate in the presence of matrix crack is higher than that in the presence of delamination provided that when the crack density is between 0.4 and 0.6 or lower than 0.4, thus matrix crack has a more critical condition to definitely occur. However, the strain energy release rate in the presence of matrix crack is lower than that in the presence of delamination for crack densities larger than 0.6, thus delamination experiences a more critical condition to certainly occur.

3.2. Bending Moment Required for the Propagation of Matrix Cracking and Induced Delamination in Cross-Ply Composite beams

The critical strain energy release rate G_{mc} is about 225 (J/m^2) in Carbon-Epoxy laminates [5]. Thus, the bending moment required for the propagation of matrix cracking and induced delamination can be calculated versus the crack density by replacing G_{mc} with G in Eqs. (6) and (7) using the proposed micromechanics model, as shown in Figs. 6 and 7 for cross-ply laminates $[90/0_2]_s$ and $[90/0]_s$, respectively.

As it can be observed in Figs. 6 and 7, the bending moment required for the propagation of matrix cracking is lower than that for the delamination in cross-ply composite laminates $[90/0_2]_s$ and $[90/0]_s$ at the crack densities equal to or lower than 0.6, thus matrix cracking propagates up to the crack density reaches 0.6.

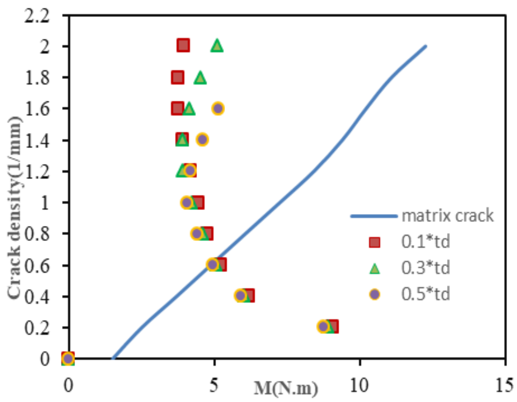

Fig. 6. Bending moment required for the propagation of matrix cracking and induced delamination in cross-ply composite laminate $[90/0_2]_s$ versus crack density.

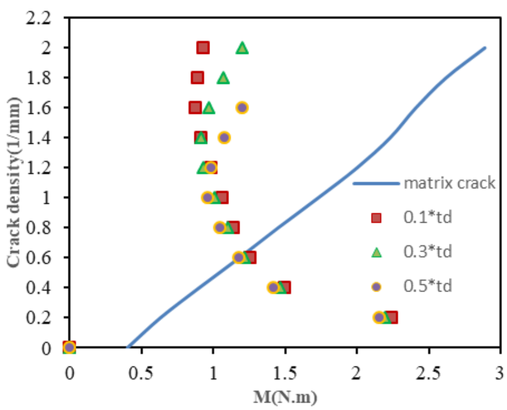

Fig. 7. Bending moment required for the propagation of matrix cracking and induced delamination in cross-ply composite laminate $[90/0]_s$ versus crack density.

The bending moment required for the delamination induction is lower at the crack densities more than 0.6, thus delamination occurs. In other words, in the investigated laminates, delamination occurs after the propagation of 3 matrix cracks in the 90° lamina, while there will be no delamination before the propagation of 3 matrix cracks. It is also observed that the bending moment required for the propagation of matrix cracking and induced delamination is larger in laminate $[90/0_2]_s$ which can be owing to its higher number of laminas.

4. Conclusions

In this article, an analytical model based on the micromechanics perspective was applied to calculate the bending moment required for the propagation of matrix cracking and induced delamination in cross-ply composite beams. To that end, a unit cell was considered from the lamina surface in the beam containing matrix cracks and delamination. The approximately obtained stress field was employed to calculate the strain energy release rate for the propagation of matrix cracking and induced delamination in the lamina surface. Later, using the finite fracture mechanics concept, the bending moment required for matrix cracking and induced delamination was calculated. To validate the proposed method, the obtained values of stiffness reduction, due to delamination, were compared with the numerical results where there was a good agreement observed between them. Above all, the proposed method could be exploited for the damage analysis of general composite beams.

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