Thermal buckling and thermal induced free vibration analysis of perforated composite plates: A mathematical model

S. Soleimanian, A. Davar, J.E. Jam*, M.R. Zamani, M. Heydari Beni
University Complex of Materials and Manufacturing Technology, Malek Ashtar University of Technology, Lavizan, Tehran, Iran

1. Introduction

The increased use of light weight engineering structures has stimulated interest in the improvement of methods for analysis of PCPs under different loadings. To design PCPs in thermal environments, considerations must be applied to the thermal induced vibration behaviour and the possibility of thermal buckling.

As a first attempt on thermal buckling of flat or initially imperfect isotropic plates, Gossard et al. [1] performed an approximate solution based on large deflection plate theory. Tauchert and Huang [2] used the Rayleigh-Ritz method to solve the thermal buckling equations for symmetric angle-ply laminated plates. Thangaratnam applied [3] the linear theory and the finite element method (FEM) to solve the thermal buckling problem of composite laminated plates. They concluded that fiber orientation, number of layers, aspect ratio and edge conditions can influence the critical buckling temperature and mode shapes significantly.

Sunt and Hsu [4] investigated the thermal buckling problem of symmetric cross-ply laminated plates using Kirchhoff deformation theory with the inclusion of transverse shear deformation in the displacement field. The critical temperature results were obtained by Navier solution procedure. They showed that the discrepancy of results obtained by the formulation with or without shear deformation components is considerable for length to thickness ratios smaller than 20. Thronton [5] carried out a research to review the temperature distributions in thin walled structures and thermal buckling analysis methods of isotropic and composite shells and plates. Whitney [6] studied expansional strain effects in laminated plates and derived formulations for bending and thermal buckling problems. Kant and Babu [7] applied two shear deformable finite element models...
based on first order shear deformation theory (FSDT) and higher order shear deformation theory (HSSDT) for thermal buckling of skew fiber reinforced composite and sandwich plates. They showed that the critical temperature values increase with increase in skew angle and it is more pronounced in thin laminates than in thick ones. Yapici [8] investigated the thermal buckling behaviour of hybrid-composite angle-ply laminated plates with an inclined crack using FEM. Pradeep and Ganesan [9] investigated thermal buckling of multi-layer rectangular viscoelastic clamped sandwich plates using FEM. Duran et al. [10] obtained thermal critical buckling temperatures of composite plates with spatial varying fiber orientations using classical lamination theory (CLPT) and FEM. Li et al. [11] applied CLPT to investigate buckling and vibro-acoustic responses of the clamped composite plates in thermal environments excited by a concentrated harmonic force. Jin et al. [12] performed digital image correlation (DIC) technique to investigate thermal buckling measurement of a circular laminated composite plate under uniform temperature distribution. By making comparisons among experimental results and those obtained by theory and nonlinear FEM analysis, they showed that DIC is promising for studying thermal buckling of composite structures in diverse fields. Bouazza et al. [13] studied thermal buckling of laminated cross-ply plates using a refined hyperbolic shear deformation theory. Cetkovic [14] proposed a layerwise displacement model to analyze the thermal buckling problem of composite plates and showed that the critical buckling temperature increases by increasing the aspect ratio of plate. Kalita and Haldar [15] reported the free vibration analysis of rectangular isotropic plates using a nine node isoparametric plate element in conjunction with first-order shear deformation theory considering the effect of rotary inertia.

Perforated structures primarily have been modeled using equivalent stiffness. A few researches [16-18] have been published to analyze structures including discontinuities using exact modeling. Takabatake [16] studied static analysis of isotropic plates with voids using the unit step function to define the structure stiffness. Takabatake used the Galerkin method to solve the differential equation of motion. Li and Cheng [17] analyzed grid stiffened composite sandwich panels with simply supported edges subjected to lateral uniform pressure. For an orthogrid stiffened plate, they considered two material regions, the cell and the surrounding ribs. Based on this concept, they modeled the grid shape in terms of Heaviside functions, which results local definition for ABD matrices. The governing equations are solved by considering only one component of displacement \( w \), so the solution is limited to symmetric sandwich lay-ups. Wilson et al. [18] has done research on elastic stability of stepped and stiffened plates. They modeled structures with variation in thickness such as single stepped, double stepped and latitudinal stiffened plates using piecewise functions for thickness.

The current study presents an analytical solution to thermal buckling and thermal induced free vibration analysis of PCPs considering uniform temperature rise and simply supported (SSSS) boundary conditions. Developing a MATLAB code, the analytical results are validated with the results obtained using ABAQUS finite element commercial software and those available in the literature. Many parametric studies have been done using the present analytical method and ABAQUS.

2. Material Modeling

A rectangular perforated plate lies in \((0,0,0) \leq (x,y,z) \leq (a,b,h)\) is considered as shown in Fig. 1a. The plate consists of a repetitive pattern of rectangular voids with equal distances from each other.

According to Fig 1, it is necessary to define a mathematical function considering material properties in orthogonal paths. For this purpose, Heaviside distribution functions are introduced by Eqs. 1(a) and 1(b) [16].

\[
H(x) = \sum_{i=1}^{m_x} \sum_{j=1}^{n_y} (\text{Heaviside}(x - x_{ci} + c/2) - \text{Heaviside}(x - x_{ci} - c/2)) \tag{1a}
\]

\[
H(y) = \sum_{i=1}^{m_x} \sum_{j=1}^{n_y} (\text{Heaviside}(y - y_{cj} + d/2) - \text{Heaviside}(y - y_{cj} - d/2)) \tag{1b}
\]
Where \((x_i, y_j)\) is the location of center of voids. The parameters \(c\) and \(d\) are the length & width of voids.

The Heaviside distribution (HD) function is given by Eq. 2 [16].

\[
HD = 1 - H(x) \cdot H(y) 
\] (2)

By plotting the HD function Fig 2., it can be observed that values of one and zero are allocated to white and black regions.

The stiffness matrix of a PCP layers can be given by:

\[
Q(x, y)^k = Q^k \cdot HD
\] (3)

where \(Q^k\) is the stiffness matrix of an orthotropic lamina given by Eq. 4 [6].

\[
Q^k = \begin{bmatrix}
E_1 & \frac{v_{12}E_2}{1-v_{12}v_{21}} & 0 \\
\frac{v_{12}E_2}{1-v_{12}v_{21}} & E_2 & 0 \\
0 & 0 & G_{12}
\end{bmatrix}
\] (4)

2. Theoretical Formulations

Linear displacement field is considered as [6]:

\[
u(x, y, z) = u_0(x, y, z) - zw_0(x, y)
\] (5)

\[
v(x, y, z) = v_0(x, y, z) - zw_0(x, y)
\] (6)

\[
w(x, y, z) = w_0(x, y)
\] (7)

Using CLPT, equilibrium equations has been derived as [6]:

\[
N_{xx} + N_{xy} + N_{iy}u_{0,xx} + N_{ij}u_{0,yy} + N_{xy}u_{0,xy} = \rho h \dddot{u}_0
\] (8)

\[
N_{xy,x} + N_{y,y} + N_{ij}v_{xx} + N_{ij}v_{0,yy} + N_{xy}v_{0,xy} = \rho h \dddot{v}_0
\] (9)

\[
M_{xx} + 2M_{xy} + M_{y,y} + N_{ij}w_{xx} + N_{ij}w_{yy} = \rho h \dddot{w}_0
\] (10)

Force and moment resultants are given by [6]:

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\
A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\
A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\
B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\
B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66}
\end{bmatrix}
\]
Where the $A$, $B$ and $D$ coefficients can be obtained by [6]:

$$A_{ij} = \sum_{k=1}^{N} (Q_{ij})_k (z_k - z_{k-1})$$  \hspace{1cm} (12)

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (Q_{ij})_k (z_k^2 - z_{k-1}^2)$$  \hspace{1cm} (13)

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (Q_{ij})_k (z_k^3 - z_{k-1}^3)$$  \hspace{1cm} (14)

And the initial uniform thermal load can be given by [6]:

$$N_{xy} = Q(x, y)_{k=3} a_{xy} k_1 h \Delta T$$  \hspace{1cm} (15)

where the parameter $\Delta T$ refers to uniform temperature difference.

The governing equilibrium equations of PCPs can be obtained by substitution of Eqs. (11) and (15) into Eqs. (8-10) as:

$$(A_{11,x} + A_{16,y})u_{0,x} + (A_{16,x} + A_{66,y})u_{0,y} + (A_{12,x} + A_{26,y})v_{0,y} + (A_{12,x} + A_{66,y})v_{0,y} - (B_{11,x} + B_{16,y})w_{0,xx} - 2(B_{16,x} + B_{66,y})w_{0,xy} - (B_{12,x} + B_{26,y})w_{0,yy} + A_{66}u_{0,yy} + A_{16}v_{0,xx} + (A_{12} + A_{66})v_{0,xy} + A_{26}v_{0,yy} - B_{11}w_{0,xxx} - 3B_{16}w_{0,xx} - (B_{12} + 2B_{66})w_{0,xy} - B_{26}w_{0,yyy} + N_{x1}^i u_{xx} + N_{y1}^i v_{xy} + N_{y1}^i v_{yy} = \rho h \ddot{u}_0$$  \hspace{1cm} (16)

$$(A_{12,x} + A_{16,y})u_{0,x} - (A_{66,x} + A_{26,y})u_{0,y} + (A_{66,x} + A_{26,y})v_{0,x} + (A_{26,x} + A_{22,y})v_{0,y} - (B_{16,x} + B_{12,y})w_{0,xx} - 2(B_{66,x} + B_{26,y})w_{0,xy} - (B_{26,x} + B_{22,y})w_{0,yy} + A_{26}u_{0,yy} + A_{66}v_{0,xx} + (A_{12} + A_{66})v_{0,xy} + A_{26}v_{0,yy} - B_{16}w_{0,xxx} - (B_{12} + 2B_{66})w_{0,xx} - 3B_{26}w_{0,xy} - B_{22}w_{0,yyy} + N_{x1}^i v_{xx} + N_{y1}^i v_{yy} + N_{y1}^i v_{xy} = \rho h \ddot{v}_0$$  \hspace{1cm} (17)

$$(B_{11,xx} + 2B_{16,xy} + B_{12,yy})u_{0,x} + (B_{16,xx} + 2B_{66,xy} + B_{22,yy})u_{0,y} + (B_{16,xx} + 2B_{66,xy} + B_{22,yy})v_{0,x} + (B_{11,xx} + 2B_{26,xy} + B_{22,yy})v_{0,y} - (D_{11,xx} + 2D_{16,xy} + D_{12,yy})w_{0,xx} - (2D_{16,xx} + 4D_{66,xy} + 2D_{26,yy})w_{0,xy} - (D_{12,xx} + 2D_{26,xy} + D_{22,yy})w_{0,yy} + 2(B_{11,x} + B_{16,y})u_{0,xx} + 2(2B_{16,x} + B_{66,y} + B_{12,y})u_{0,xy} + 2(B_{66,x} + B_{26,y})u_{0,yy} + 2(B_{16,x} + B_{66,y})v_{0,xx} + 2(B_{12,x} + B_{26,y} + B_{66,x})v_{0,yy} + 2(2B_{26,x} + B_{22,y})v_{0,yy} - 2(D_{11,x} + D_{12,y} + D_{66,x} + 6D_{66,y})w_{0,xx} - 2(D_{12,x} + 6D_{26,y} + 2D_{66,x})w_{0,xy} - 2(D_{22,x} + D_{66,x})w_{0,yy} + 2(2B_{66} + B_{12})u_{0,xy} + 2B_{26}u_{0,yy} + B_{16}v_{0,xx} + (B_{12} + 2B_{66})v_{0,xx} + 3B_{26}v_{0,xy} + 3B_{26}v_{0,yy} + 2B_{22}v_{0,yy} - D_{11}w_{0,xxx} - 4D_{16}w_{0,xx} - (2D_{12} + 4D_{66})w_{0,xy} - 4D_{26}w_{0,xyy} - 4D_{26}w_{0,yyy} - D_{22}w_{0,yyyy} - N_{x1}^i w_{xx} + N_{y1}^i w_{xy} + N_{y1}^i w_{yy} + N_{y1}^i w_{xy} = \rho h \ddot{w}_0$$  \hspace{1cm} (18)

where the second indices are devoted to local derivatives.

By considering displacement functions corresponding to SSSS edge conditions as [19]:

$$u_0(x, y) = U_{mn} \cos \left( \frac{m \pi}{a} x \right) \sin \left( \frac{n \pi}{b} y \right) T(t)$$  \hspace{1cm} (19)

$$v_0(x, y) = V_{mn} \sin \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) T(t)$$  \hspace{1cm} (20)

$$w_0(x, y) = W_{mn} \sin \left( \frac{m \pi}{a} x \right) \sin \left( \frac{n \pi}{b} y \right) T(t)$$  \hspace{1cm} (21)

and using Galerkin method, the system of PDEs given by Eqs. (16-18) lead to Eqs. (22) and (23) for thermal buckling ($T(t)=1$) and thermal induced free vibration ($T(t) = e^{\omega t}$) analyses, respectively. The parameters $\omega$ and $t$ refer to frequency and time, respectively.

For thermal buckling and thermal induced free vibration analyses, the eigenvalue problems given by Eq. (22) and (23) [19] have been solved.
\[(k^e + k^T)\{\delta\} = 0 \quad (22)\]
\[m\{\ddot{\delta}\} + (k^e + k^T)\{\delta\} = 0 \quad (23)\]

where \(k^e\) and \(k^T\) refer to elastic and thermal stiffness matrix coefficients, respectively. \(M\) is the mass matrix and \(\delta\) is the displacement vector. \(k^e\), \(k^T\) and \(M\) matrices are expanded in Appendices (1-3).

4. FEM Modeling

In order to verify the analytical results, FEM models are performed using ABAQUS to simulate thermal buckling and thermal induced free vibration problems.

The FEM meshed model Fig 3. is produced using S4R elements.

![Fig 3. FEM Meshed model.](image)

For a PCP considering material and geometry properties according to Tables 1 and 2, mesh convergence is reported for critical temperature difference and fundamental frequency in Table 3.

<p>| Table 1. Material properties for the PCP [20] |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>(E_1) (GPa)</th>
<th>(E_2) (GPa)</th>
<th>(G_{12}) (GPa)</th>
<th>(\alpha_1) (1/°C)</th>
<th>(\alpha_2) (1/°C)</th>
<th>(\rho) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.6</td>
<td>8.27</td>
<td>4.14</td>
<td>0.26</td>
<td>8.6 x 10⁻⁵</td>
<td>22.1 x 10⁻⁵</td>
</tr>
</tbody>
</table>

<p>| Table 2. Geometry properties for the PCP |
|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>(a) (mm)</th>
<th>(b) (mm)</th>
<th>(h) (mm)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>200</td>
<td>4</td>
<td>10%</td>
</tr>
</tbody>
</table>

As shown in Table 3, the enough value for element edge length to plate edge length ratio is achieved 0.00225.

<p>| Table 3. Mesh convergence for the PCP |
|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>element size to plate length ratio</th>
<th>(\Delta T_c) (°C)</th>
<th>Fundamental frequency (Hz)</th>
<th>A partial view of the meshed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>38.228</td>
<td>258.62</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>37.936</td>
<td>255.42</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>38.038</td>
<td>255.58</td>
<td></td>
</tr>
<tr>
<td>0.00225</td>
<td>38.072</td>
<td>255.18</td>
<td></td>
</tr>
<tr>
<td>0.00125</td>
<td>38.067</td>
<td>255.06</td>
<td></td>
</tr>
</tbody>
</table>

5. Validation

The accuracy of analytical solutions for buckling and free vibration are compared with those reported in literature.

For an isotropic plate \((\nu=0.3, \alpha=7.4 \times 10^{-6} \text{ 1/°C})\) considering different values of \(a/h\) ratio, the accuracy of the present analytical and ABAQUS methods for thermal buckling are checked through comparisons with the Ref. [14] as shown in Table 4. A maximum discrepancy of 1.4% is obtained between the present analytical and Cetkovic results. So that, the present analytical method can be used reliably for isotropic plates with \(a/h\) ratios bigger than or equal to 20.

Considering a composite plate (layup: \([0/90]\) s, \(E_1/E_2=20\), \(G_{12}/E_2=G_{13}/E_2=G_{23}/E_2=0.5\), \(\alpha_1/\alpha_2=0.25\), \(\nu_{12}=\nu_{13}=\nu_{23}=0.25\)), the second validation is conducted to check the accuracy of the present analytical solution with the analytical method developed by Bouzza [13]. As shown in Fig 4., the ABAQUS results are closer to the Ref. [13] values in comparison with the analytical results. The maximum discrepancy between the present analytical results and Ref. [13] is about 5.9% which corresponds to \(a/h=2.5\).

| Table 4. Comparisons of critical temperature difference results (°C) for an isotropic plate \((\nu=0.3, \alpha=7.4 \times 10^{-6} \text{ 1/°C})\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(\Delta T_c\) (°C) | Fundamental frequency (Hz) | A partial view of the meshed model |
|-----------------|-----------------|-----------------|-----------------|
| 38.228          | 258.62          | |
| 37.936          | 255.42          | |
| 38.038          | 255.58          | |
| 38.072          | 255.18          | |
| 38.067          | 255.06          | |
Table 5 shows the results of Non-dimensional frequency \( \omega^* = \omega a^2 \sqrt{\rho h / D(2,2)} \) with corresponding mode shapes for an isotropic plate with singular central perforation \( (\nu=0.3, c/a=0.2) \). Table 5 include the results obtained by Kalita and Haldar [15] and the present study results and the maximum discrepancy is 3.08%. The present ABAQUS results are in closer agreement with the Ref. [15], because both methods are developed based on nonlinear displacement field.

**Table 5.** Non-dimensional frequency \( \omega^* = \omega a^2 \sqrt{\rho h / D(2,2)} \) results for isotropic plate with singular central perforation \( (\nu=0.3, c/a=1/5) \)

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>Mode(1,1)</th>
<th>Mode(1,2)</th>
<th>Mode(2,1)</th>
<th>Mode(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kalita (2016) [15]</td>
<td>19.1</td>
<td>47.496</td>
<td>47.496</td>
<td>76.25</td>
</tr>
<tr>
<td>Present ABAQUS</td>
<td>19.128</td>
<td>47.6649</td>
<td>47.6649</td>
<td>76.39</td>
</tr>
</tbody>
</table>

![Graph](image-url)
6. Results and Discussion

To evaluate the thermal buckling behaviour of PCPs efficiently, some case studies are carried out in sections 5.1 to 5.7.

For studied perforated plates, the number of voids (mx=ny) are considered equal to 20 and the volume fraction of voids is defined as:

$$R = \frac{m_x n_y c \times d}{a \times b} \times 100$$

(24)

It is assumed that R=10%, unless for sections 5.3 and 5.7 where different values of the parameter R is considered. The h/a ratio is taken 0.02 for all studies.

In sections 6.2, 6.5 and 6.6, the material is considered as given in Table 1.

6.1 Effect of elastic modulus and thermal expansion coefficient ratios (E2/E1 and α2/α1) on the critical temperature difference

In order to study the effect of parameters E2/E1 and α2/α1 on thermal buckling of PCPs, the material properties are given in Table 6. The stacking sequence is assumed to be [0]. As shown in Fig. 5, by increasing the values of E2/E1 and α2/α1 ratios, the critical temperature difference decreases. A maximum discrepancy of 5.82% is achieved between ABAQUS and analytical results where E2/E1=0.2 and α2/α1=1.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ABAQUS</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2/E1</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>α2/α1</td>
<td>1.2</td>
<td>1.25</td>
</tr>
</tbody>
</table>

6.2 Comparison between the critical temperature difference of PCP and SWCCP

The critical temperature difference of PCP and the same weight monolithic composite plate (SWCCP) is indicated in Table 7. The stacking sequence is considered to be [0/90]s. As the volume fraction of the voids increases, the PCP demonstrates higher resistance to thermal buckling than the SWCCP. It can be observed that, a PCP can improve the critical temperature difference four times higher than SWCCP at R=50%. Figs 6(a) and 6(b) show the critical thermal buckling modes for PCP and SWCCP, respectively.

<table>
<thead>
<tr>
<th>R (%)</th>
<th>t (mm)</th>
<th>(ΔT_c) (Analytical) (°C)</th>
<th>(ΔT_c) (ABAQUS) (°C)</th>
<th>t (mm)</th>
<th>(ΔT_c) (Analytical) (°C)</th>
<th>(ΔT_c) (ABAQUS) (°C)</th>
<th>Analytical</th>
<th>ABAQUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>38.2243</td>
<td>38.072</td>
<td>3.6</td>
<td>30.9689</td>
<td>30.531</td>
<td>1.23</td>
<td>1.25</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>38.2153</td>
<td>38.216</td>
<td>3.2</td>
<td>24.4693</td>
<td>24.168</td>
<td>1.56</td>
<td>1.58</td>
</tr>
</tbody>
</table>
6.3 Effect of elastic modulus ratio (E2/E1) on the thermal induced fundamental frequency

In order to study the effect of E2/E1 ratio on the thermal induced fundamental frequency of PCPs, the material properties are considered as given in Table 8. The stacking sequence is assumed to be [0]. As shown in Fig. 7, by increasing the E2/E1 ratio, the thermal induced fundamental frequencies demonstrate ascending and descending behaviour before and after the intersection point at the coordinate (32.28, 272.67). A maximum discrepancy of 8.27% is achieved between ABAQUS and analytical results at E2/E1 = 0.6.

Table 8. Material properties to study the effect of E2/E1 ratio

<table>
<thead>
<tr>
<th>E1 (GPa)</th>
<th>E2 (GPa)</th>
<th>G12 (GPa)</th>
<th>α12 (1/°C)</th>
<th>α1 (1/°C)</th>
<th>E2 (GPa)</th>
<th>E1 (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>variable</td>
<td>15.873</td>
<td>0.26</td>
<td>10^{-6}</td>
<td>0.26</td>
<td>10^{-6}</td>
</tr>
</tbody>
</table>

6.4 Effect of thermal expansion coefficient ratio (α2/α1) on the thermal induced fundamental frequency

In order to study the effect of α2/α1 ratio on the thermal induced fundamental frequency of PCPs, the material properties are considered as shown in Table 9. The stacking sequence is assumed [0]. As shown in Fig. 8, by increasing the α2/α1 ratio, the thermal induced fundamental frequency is decreased. A maximum discrepancy of 9.14% is observed between ABAQUS and analytical results for T=0 and α2/α1=1.25.

6.5 Effect of stacking sequence on the thermal induced fundamental frequency

In order to study the effect of the stacking sequence on the free vibration behaviour of PCPs, [0/90] s and [0/90] stacking sequences are considered. According to Fig. 9, the thermal induced fundamental frequency corresponded to the symmetric stacking sequence is bigger than the unsymmetric one. It can be concluded that the PCP with symmetric stacking sequence vibrates with higher frequency. Furthermore, it can be observed that the symmetric...
structure buckles at higher temperature difference than the unsymmetric one.

![Graph showing analytical results for thermal induced fundamental frequency of PCPs considering different α_2/α_1 ratios.](image)

**Fig 9.** Analytical results for thermal induced fundamental frequency of PCPs considering different α_2/α_1 ratios.

### 6.6 Comparison between thermal induced fundamental frequency of PCP and SWCCP

The thermal induced fundamental frequency of PCP and SWCCP is indicated in Table 10 for void volume fraction R=10%. The fundamental frequency of the PCP decreases with lower rate in comparison with SWCCP as the temperature difference increases.

### Conclusion

A mathematical approach to thermal buckling and thermal induced free vibration analyses of perforated composite plates (PCPs) are described and discussed in this research. The present results are compared to the previously published researches [13-15] and they were found to be in close agreement.

According to results, several concluding remarks are listed as follows:

- By increasing the E2/E1 and α2/α1 ratios, the critical temperature difference of orthotropic perforated plate is decreased.

- Using the same material properties, the PCP (R=50%) is 4 times more resistant for thermal buckling than the monolithic composite plate of the same weight.

- By increasing the E2/E1 ratio, the thermal induced fundamental frequency of the orthotropic perforated plate demonstrate ascending and descending behaviour before and after a specific intersection point.

- It can be concluded that the PCP with symmetric stacking sequence vibrates with higher frequency and buckles at higher critical temperature difference rather than the PCP with unsymmetric layup.

- Using the same material properties, variation of the fundamental frequency of the PCP (at R=10%) is less than the monolithic composite plate of the same weight by increasing the temperature.

### Appendix 1

\[
k_{11}^e = \int_0^b \int_0^a \left( (A_{11,x} + A_{16,y})u_{0,x} + (A_{16,x} + A_{66,y})u_{0,y} 
+ A_{16}u_{0,xx} + (A_{12} + A_{66})u_{0,xy} 
+ A_{26}u_{0,yy} \right) U_{mn}(x, y) \, dx \, dy
\]

(A.1)

\[
k_{12}^e = \int_0^b \int_0^a \left( (A_{16,x} + A_{66,y})v_{0,x} 
+ (A_{12} + A_{66})v_{0,xx} + (A_{12} + A_{66})v_{0,xy} 
+ A_{26}v_{0,yy} \right) V_{mn}(x, y) \, dx \, dy
\]

(A.2)
\[ k_{13}^e = \int_0^b \int_0^a \left( (A_{1.6} + A_{1.2})u_{0.x} ight) \text{d}x \text{d}y \]

\[ - (A_{1.6} + A_{2.6})u_{0.y} + A_{1.6}u_{0.xx} + A_{1.2} + A_{6.6}u_{0.xy} + A_{26}u_{0.yy} \]

\[ k_{73}^e = \int_0^b \int_0^a \left( (A_{66.6} + A_{2.6})v_{0.x} \right) \text{d}x \text{d}y \]

\[ - (A_{66.6} + A_{2.6})v_{0.y} + A_{66.6}v_{0.xx} + 2A_{66}v_{0.xy} + A_{22}v_{0.yy} \]

\[ k_{73}^s = \int_0^b \int_0^a \left( -(B_{1.6} + B_{1.2})w_{0.xx} \right) \text{d}x \text{d}y \]

\[ - 2(B_{1.6} + B_{1.2})w_{0.xy} - (B_{1.6} + B_{1.2})w_{0.yy} \]

\[ - B_{16}w_{0.xxx} - (B_{12} + 2B_{66})w_{0.xxx} - 3B_{26}w_{0.xyy} \]

\[ - B_{22}w_{0.yyy} W_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{12}^e = \int_0^b \int_0^a \left( (B_{16.x} + 2B_{66.xy} + B_{26.yy})v_{0.x} \right) \text{d}x \text{d}y \]

\[ + (B_{12.xx} + B_{26.xx} + B_{22.yy})v_{0.y} + 2(B_{16.x} + B_{66.yy})v_{0.xx} + 2(B_{12.xx} + 2B_{26.yy} + B_{66.xx})v_{0.xy} + (B_{12} + 2B_{66})v_{0.xyy} + B_{22}v_{0.yyy} \]

\[ (A.7) \]

Appendix 2

\[ k_{11}^T = \int_0^a \int_0^b (N_{11}^i u_{xx} + N_{11}^i u_{yy}) U_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{12}^T = \int_0^a \int_0^b N_{12}^i u_{xy} V_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{21}^T = \int_0^a \int_0^b N_{21}^i u_{xy} U_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{12}^j = \int_0^a \int_0^b (N_{11}^i v_{xx} + N_{11}^i v_{yy}) V_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{22}^j = \int_0^a \int_0^b (N_{21}^i u_{xx} + N_{21}^i u_{yy}) W_{mn}(x, y) \text{d}x \text{d}y \]

\[ k_{22}^j = 0 \]

\[ k_{23}^j = 0 \]

\[ k_{33}^j = 0 \]
Appendix 3

\[ M_{11} = \int_0^a \int_0^b \int_{-h/2}^{h/2} \rho U_{\mn}(xy) dxdy \]
\[ M_{12} = 0 \]
\[ M_{13} = 0 \]
\[ M_{21} = 0 \]
\[ M_{22} = \int_0^a \int_0^b \int_{-h/2}^{h/2} \rho V_{\mn}(xy) dxdy \]
\[ M_{31} = 0 \]
\[ M_{32} = 0 \]
\[ M_{33} = \int_0^a \int_0^b \int_{-h/2}^{h/2} \rho W_{\mn}(xy) dxdy \]

References