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Elasto-dynamic Response Analysis of a Curved Composite Sandwich Beam Subjected to the Loading of a Moving Mass

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KEYWORDS

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ABSTRACT

In this paper, the dynamic response of a simply - supported relatively thick composite sandwich curved beam under a moving mass is investigated. In contrast to previous works, the geometry of beam is considered to be in a curved form. Moreover, the rotary inertia and the transverse shear deformation are also considered in the present analysis. The governing equations of the problem are derived using Hamilton's principle. Then, the obtained partial differential equations are transformed to the ordinary differential equations with time varying coefficients, using the modal analysis method. Fourth-order Runge-Kutta method is applied to solve the ordinary differential equations in an analytical – numerical form. The obtained results are validated by the results existed in the literature. Performing a thorough parametric study, the effects of some important parameters such as the mass and the velocity of moving mass, the radius of curvature of the beam, the core thickness to the total thickness ratio and the stacking sequences of the face sheets on the dynamic response are investigated. It is observed that increasing the mass and the velocity of moving mass and the radius of curvature of beam, result in an increase, decrease and increase of the dynamic deflection of curved beam, respectively.

1. Introduction

The analysis of dynamic behavior of bridges subjected to moving forces or moving masses is one of the most important problems facing structural and design engineers. This is an old challenge in structural dynamics. Since, recently, the speed and weight of the commercial vehicles have been increased, so, the bridge structures are fabricated much lighter due to the economical requirements. Therefore, these structures experience severe vibrations and dynamic stresses, which consequently are much more than the corresponding static ones.

It is evident that the inertial effects of a heavy vehicle moving on an elastic structure, especially at high speeds, are very important [1]. Moreover, the separation between the mass and the supporting structure may happen for greater vehicle to beam mass ratio [2]. In recent years, the traditional heavy beams made of simple materials are gradually being replaced by stronger light composite ones. The use of composite materials has remarkably increased in different engineering applications because of their high strength, stiffness and suitable failure characteristics.

As the equation of motion for the moving mass problem contains time-varying coefficients, a closed form solution is not available. Hence, various approximate techniques have been utilized to solve the problem [3-5]. Generally, the analysis of the moving loads on the bridges goes back to the 19th century, when railroad construction was first started. Researchers still work on this subject, especially due to the development of the numerical techniques for solving the complicated differential equations. Since the middle of the 19th century, the problem of fluctuation of bridges under moving loads has interested many engineers [6]. Timoshenko [7] studied the problem of a pulsating load passing over a bridge, while Inglis [8] conducted an analysis on trains crossing a bridge and studied many important factors.

Stanisic and Hardin [9] performed the dynamic analysis of a simply supported beam subjected to a moving mass. A comprehensive study on the subject of the vibration of structures causing by moving loads has been

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presented by Fryba [10]. Esmailzadeh and Ghorashi [11] investigated the vibration of an Euler-Bernoulli beam under uniform partially distributed moving mass. Yang et al. [12] presented a general theory for treating the vibration of a horizontally curved beam subjected to moving masses, each of which was simulated as a gravitational force and a centrifugal force. The problem was solved in an analytical but approximate manner considering the contribution of the first mode of vibration. Wu and Chiang [13] solved the forced vibration responses of a horizontally curved beam subjected to a moving load using the Newmark direct integration method. Since, for the curved beams studied, the in-plane responses and the out-of-plane responses were uncoupled, the in-plane behaviors of the curved beam were neglected. In addition to the curved beam element and the consistent-mass model, they also used the conventional straight beam element and the lumped-mass model to perform the free and forced vibration analyses of the curved beams. Lou et al. [14] studied a Timoshenko beam subjected to a moving mass using finite element method. Nikkhoo et al. [15] studied an Euler–Bernoulli beam under the excitation of moving mass. They obtained an approximate formulation to the problem by limiting the inertial effect of the moving mass merely to the vertical component of acceleration. Kahya and Mosallam [16] obtained approximate analytical solution for the dynamic response of composite sandwich beams subjected to moving mass. They investigated the effects of the lamina thickness and the fiber orientation on the beam deflection and the contact force between the beam and the mass. Dai and Ang [17] presented an analytical solution to the steady-state response of a curved beam resting on a viscously damped foundation and subjected to a single or sequence of moving loads. They also carried out a computational study of the problem using the moving element method based on the piecewise straight beam elements. Chen et al. [18] investigated the nonlinear dynamic responses of the fiber-metal laminated beam resting on a tensionless elastic foundation and subjected to a moving harmonic load and thermal load. The nonlinear governing equations were derived using Hamilton principle and solved by finite difference method, Newmark method and Newton-Raphson method.

Sheng and Wang [19] determined the nonlinear dynamic responses of structures under moving loads. The load was modeled as a four degrees-of-freedom system with linear suspensions and tires flexibility and the structure was considered as a simply supported Euler–Bernoulli beam. Li and Ren [20] derived the three-directional analytical solutions for responses of curved beams induced by moving loads, involving vertical, torsional, radial and axial motions. Taking a curved bridge under passage of a vehicle as an example, the influences of system parameters, such as vehicle speed, braking acceleration, bridge curve radius, bridge span and bridge deck elastic modulus, on bridge midpoint vibration were explored. Szyłko-Bigus et al. [21] studied the dynamic behavior of a Rayleigh multi-span uniform continuous beam system traversed by a constant moving force or a uniformly distributed load. The velocity of the load was considered as constant. The problem was solved using an analogue of the static force method and instead of an algebraic set of equations, a set of Volterra integral equations.

In the present paper, the dynamic response of a simply supported curved composite sandwich beam under a moving mass is analyzed. In the present research, the geometry of the beam is considered to be in a curved form. The displacements field is defined based on Timoshenko beam theory in polar coordinate system. Hamilton's principle is employed to derive the governing equations of problem. Using the modal analysis method, the obtained partial differential equations are transformed to ordinary differential equations with time varying coefficients. The ordinary differential equations are solved in an analytical – numerical form by fourth order Runge-Kutta method. The validity of the present analysis is confirmed through comparison of the present results and the results existed in the literature. Also, the effects of different parameters such as the mass and the velocity of the moving mass, the radius of the curvature of the beam, the core thickness to the total thickness ratio and the stacking sequences of the face sheets on the dynamic response are studied.

2. Analytical Formulations

As shown in Fig. (1), a curved composite sandwich beam with simply supported boundary conditions subjected to a moving mass is considered.

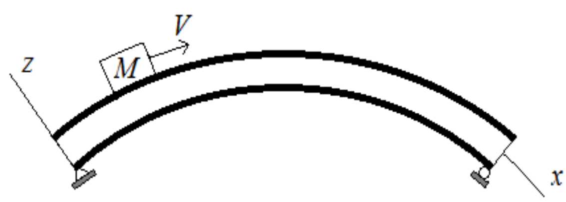

Fig. 1. Schematic view of a simply supported curved composite sandwich beam under a moving mass

It is assumed that the mass moves with a constant velocity on the top surface of the beam. Also, the damping of the beam is neglected. Based on Timoshenko beam theory in polar coordinate system the displacements field is defined as:

$$\begin{aligned} u(\theta,z,t) &= u_0(\theta,t) + z\phi(\theta,t) \\ w(\theta,z,t) &= w_0(\theta,t) \end{aligned} \tag{1}$$

where u and w are the displacements in x and z directions respectively and $\emptyset$ is the rotation about axis y.

The strain – displacement relations are:

$$\begin{aligned} \varepsilon_{\theta\theta} &= \left(\frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{w}{R}\right) \\ \gamma_{\theta\theta} &= \left(\frac{1}{R}\frac{\partial w}{\partial \theta} - \frac{u}{R}\right) + \frac{\partial u}{\partial z} \\ \varepsilon_{zz} &= \frac{\partial w}{\partial z} \end{aligned} \tag{2}$$

Substituting Eqs. (1) intoEqs. (2), the strain – displacement relations are obtained as:

$$\begin{aligned} \varepsilon_{\theta\theta} &= \frac{1}{R}\left(\frac{\partial u_0}{\partial \theta} + z\frac{\partial \Phi}{\partial \theta} + w_0\right) \\ \gamma_{\theta z} &= \frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{u_0}{R} - \frac{z}{R}\Phi + \Phi \\ \varepsilon_{zz} &= \frac{\partial w_0}{\partial z} \end{aligned} \tag{3}$$

For a thin beam, the value of z/R is small and can be neglected. So it can be written as:

$$\gamma_{\theta z} = \frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{u_0}{R} + \Phi \tag{4}$$

To obtain the governing equation of the problem, Hamilton's principle is employed. So it can be written as:

$$\delta\int_0^t (\mathrm{T} - \mathrm{U} - \mathrm{V})\ dt = 0 \tag{5}$$

In which T is the kinetic energy, U the strain energy and V the potential energy. The variation of the kinetic energy functional is expressed as:

$$\begin{aligned} \int_0^t \delta T &= -\int_0^t\int_0^{\varphi_0}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^b \rho\left(\ddot{u}\,\delta u + \ddot{w}\,\delta w\right) R\,dy\ dz\ d\theta\ dt + \\ &\int_0^{\varphi_0}\int_0^t \left[MV^2 w'' + 2VM\dot{w}' + M\ddot{w}\right] \times \delta_d\left(R\theta - Vt\right)\delta w_0 R\,d\theta\,dt \end{aligned} \tag{6}$$

where ρ is the density, R the radius of the curvature, M and V the mass and the velocity of the moving mass, respectively, $w' = \dfrac{dw}{dx}$ and $\dot{w} = \dfrac{dw}{dt}$. Also, the term $MV^2 w''$ is the centrifugal force which reduces somewhat to the bending stiffness of the beam. The term $2VM\dot{w}'$ is the carioles force which plays the role of a damper for the system. The term $M\ddot{w}$ is the inertia force of the moving mass in the radial direction. The operator δ_d is Dirac Delta function. t is the time and b and φ_0 the width and the circumferential length of the curved beam, respectively.

Substituting Eqs. (1) into Eqs. (6) and calculating the integrals with respect to y and z coordinates gives:

$$\begin{aligned} \int_0^t \delta T &= -\int_0^t\int_0^{\varphi_0} R[\left(I_1\ddot{u}_0 + I_2\ddot{\Phi}\right)\delta u_0 + \left(I_2\ddot{u}_0 + I_3\ddot{\Phi}\right)\delta\Phi + \\ &\left\{I_1\ddot{w}_0 + [MV^2 w'' + 2VM\dot{w}' + M\ddot{w}]\delta_d\left(R\theta - Vt\right)\right\}\delta w_0]d\theta dt \end{aligned} \tag{7}$$

where I_1, I_2 and I_3 are the mass moments of inertia defined as:

$$I_1 = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz,\ I_2 = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho z dz,\ I_3 = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho z^2 dz \tag{8}$$

The strain energy functional is written as:

$$\begin{aligned} \int_0^t \boldsymbol{\delta U} &= \int_0^t \int_0^{\varphi_0} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b [\boldsymbol{\sigma_\theta \delta \varepsilon_{\theta\theta}} \\ &+ \boldsymbol{\tau_{\theta z} \delta \gamma_{\theta z}}]\boldsymbol{Rdydzd\theta dt} \end{aligned} \tag{9}$$

Substituting Eqs. (3) and (4) into Eq. (9), the strain energy functional can be written as:

$$\begin{aligned} \int_0^t \delta U &= \int_0^t\int_0^{\varphi_0}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^b [\frac{\sigma_\theta}{R}\delta\left(\frac{\partial u_0}{\partial \theta} + z\frac{\partial \Phi}{\partial \theta} + w_0\right) + \\ &\tau_{\theta z}\delta\left(\frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{u_0}{R} + \Phi\right)]R\,dydzd\theta dt \end{aligned} \tag{10}$$

Calculating the integrals with respect to y and z coordinates gives:

$$\begin{aligned} \int_0^t \delta U &= \int_0^t\int_0^{\varphi_0} [N_{\theta\theta}\delta\left(\frac{\partial u_0}{\partial \theta}\right) + M_{\theta\theta}\delta\left(\frac{\partial \Phi}{\partial \theta}\right) + N_{\theta\theta}\delta w_0 + \\ &N_{\theta z}\delta\left(\frac{\partial w_0}{\partial \theta}\right) - N_{\theta z}\delta u_0 + RN_{\theta z}\delta\Phi]d\theta dt \end{aligned} \tag{11}$$

in which $N_{\theta\theta}$ and $N_{\theta z}$ are the force resultants and $M_{\theta\theta}$ is the moment resultant defined as:

$$\begin{aligned} N_{\theta\theta} &= A_{11}\left(\frac{1}{R}\frac{\partial u_0}{\partial \theta} + \frac{w_0}{R}\right) + \frac{B_{11}}{R}\frac{\partial \Phi}{\partial \theta} \\ N_{\theta z} &= E_{11}\left(\frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{u_0}{R} + \Phi\right) \\ M_{\theta\theta} &= B_{11}\left(\frac{1}{R}\frac{\partial u_0}{\partial \theta} + \frac{w_0}{R}\right) + \frac{D_{11}}{R}\frac{\partial \Phi}{\partial \theta} \end{aligned} \tag{12}$$

After some mathematical manipulations, the strain energy functional is obtained as:

$$\begin{aligned} \int_0^t \delta U = \\ \int_0^t \int_0^{\varphi_0} [-\frac{1}{R}A_{11}\frac{\partial^2 u_0}{\partial \theta^2}\delta u_0 \\ - \frac{1}{R}A_{11}\frac{\partial w_0}{\partial \theta}\delta u_0 \\ - \frac{1}{R}B_{11}\frac{\partial^2 \Phi}{\partial \theta^2}\delta u_0 \end{aligned} \tag{13}$$

$$-\frac{1}{R}B_{11}\frac{\partial^2 u_0}{\partial\theta^2}\delta\Phi-\frac{1}{R}B_{11}\frac{\partial w_0}{\partial\theta}\delta\Phi -\frac{1}{R}D_{11}\frac{\partial^2\Phi}{\partial\theta^2}\delta\Phi +\frac{1}{R}A_{11}\frac{\partial u_0}{\partial\theta}\delta w_0+\frac{1}{R}A_{11}w_0\delta w_0 +\frac{1}{R}B_{11}\frac{\partial\Phi}{\partial\theta}\delta w_0 -\frac{1}{R}E_{11}\frac{\partial^2 w_0}{\partial\theta^2}\delta w_0+\frac{1}{R}E_{11}\frac{\partial u_0}{\partial\theta}\delta w_0 -E_{11}\frac{\partial\Phi}{\partial\theta}\delta w_0 -\frac{1}{R}E_{11}\frac{\partial w_0}{\partial\theta}\delta u_0+\frac{1}{R}E_{11}u_0\delta u_0-E_{11}\Phi\delta u_0 +E_{11}\frac{\partial w_0}{\partial\theta}\delta\Phi-E_{11}u_0\delta\Phi +RE_{11}\Phi\delta\Phi]d\theta dt$$

The potential energy functional is:

$$\int_0^t \delta V\,dt=-\int_0^t\int_0^{\varphi_0} Mg\,\delta_d\left(R\theta-Vt\right)\delta w_0 R\,d\theta dt \tag{14}$$

Using the obtained expressions for the kinetic energy, the strain energy and the potential energy functionals, the Hamilton principle takes the form:

$$\begin{aligned}&\int_0^t\int_0^{\varphi_0}[R[\left(I_1\ddot{u}_0+I_2\ddot{\Phi}\right)\delta u_0+\left(I_2\ddot{u}_0+I_3\ddot{\Phi}\right)\delta\Phi+\\ &\left\{I_1\ddot{w}_0+[MV^2w''+2VM\dot{w}'+M\ddot{w}]\delta_d\left(R\theta-Vt\right)\right\}\delta w_0]\\ &-\frac{1}{R}A_{11}\frac{\partial^2u_0}{\partial\theta^2}\delta u_0-\frac{1}{R}A_{11}\frac{\partial w_0}{\partial\theta}\delta u_0-\frac{1}{R}B_{11}\frac{\partial^2\Phi}{\partial\theta^2}\delta u_0\\ &-\frac{1}{R}B_{11}\frac{\partial^2u_0}{\partial\theta^2}\delta\Phi-\frac{1}{R}B_{11}\frac{\partial w_0}{\partial\theta}\delta\Phi-\frac{1}{R}D_{11}\frac{\partial^2\Phi}{\partial\theta^2}\delta\Phi\\ &+\frac{1}{R}A_{11}\frac{\partial u_0}{\partial\theta}\delta w_0+\frac{1}{R}A_{11}w_0\delta w_0+\frac{1}{R}B_{11}\frac{\partial\Phi}{\partial\theta}\delta w_0\\ &-\frac{1}{R}E_{11}\frac{\partial^2w_0}{\partial\theta^2}\delta w_0+\frac{1}{R}E_{11}\frac{\partial u_0}{\partial\theta}\delta w_0-E_{11}\frac{\partial\Phi}{\partial\theta}\delta w_0\\ &-\frac{1}{R}E_{11}\frac{\partial w_0}{\partial\theta}\delta u_0+\frac{1}{R}E_{11}u_0\delta u_0-E_{11}\Phi\delta u_0+E_{11}\frac{\partial w_0}{\partial\theta}\delta\Phi-\\ &E_{11}u_0\delta\Phi+RE_{11}\Phi\delta\Phi-RMg\,\delta_d\left(R\theta-Vt\right)\delta w_0]d\theta dt=0\end{aligned}\tag{15}$$

Eq. (15) can be simplified to:

$$\begin{aligned}&\int_0^t\int_0^{\varphi_0}\{[RI_1\ddot{u}_0+RI_2\ddot{\Phi}-\frac{1}{R}A_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}A_{11}\frac{\partial w_0}{\partial\theta}-\frac{1}{R}B_{11}\frac{\partial^2\Phi}{\partial\theta^2}-\\ &\frac{1}{R}E_{11}\frac{\partial w_0}{\partial\theta}+\frac{1}{R}E_{11}u_0-E_{11}\Phi]\delta u_0+[RI_2\ddot{u}_0+RI_3\ddot{\Phi}-\\ &\frac{1}{R}B_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}B_{11}\frac{\partial w_0}{\partial\theta}-\frac{1}{R}D_{11}\frac{\partial^2\Phi}{\partial\theta^2}+E_{11}\frac{\partial w_0}{\partial\theta}-\\ &E_{11}u_0+RE_{11}\Phi]\delta\Phi+[RI_1\ddot{w}_0+R[MV^2w''+2VM\dot{w}'+\\ &M\ddot{w}-Mg]\delta_d\left(R\theta-Vt\right)+\frac{1}{R}A_{11}\frac{\partial u_0}{\partial\theta}+\frac{1}{R}A_{11}w_0+\\ &\frac{1}{R}B_{11}\frac{\partial\Phi}{\partial\theta}-\frac{1}{R}E_{11}\frac{\partial^2w_0}{\partial\theta^2}+\frac{1}{R}E_{11}\frac{\partial u_0}{\partial\theta}-E_{11}\frac{\partial\Phi}{\partial\theta}]\delta w_0\}d\theta dt=0\end{aligned}\tag{16}$$

The circumferential displacement u is very small compared to the transverse displacement w , so neglecting the corresponding terms, Eq. (16) becomes:

$$\begin{aligned}&\int_0^t\int_0^{\varphi_0}\{[RI_2\ddot{\Phi}-\frac{1}{R}A_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}A_{11}\frac{\partial w_0}{\partial\theta}-\frac{1}{R}B_{11}\frac{\partial^2\Phi}{\partial\theta^2}-\frac{1}{R}E_{11}\frac{\partial w_0}{\partial\theta}-\\ &E_{11}\Phi]\delta u_0+[RI_3\ddot{\Phi}-\frac{1}{R}B_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}B_{11}\frac{\partial w_0}{\partial\theta}-\frac{1}{R}D_{11}\frac{\partial^2\Phi}{\partial\theta^2}+\\ &E_{11}\frac{\partial w_0}{\partial\theta}+RE_{11}\Phi]\delta\Phi+[RI_1\ddot{w}_0+R[MV^2w''+2VM\dot{w}'+\\ &M\ddot{w}-Mg]\delta_d\left(R\theta-Vt\right)+\frac{1}{R}A_{11}w_0+\frac{1}{R}B_{11}\frac{\partial\Phi}{\partial\theta}-\\ &\frac{1}{R}E_{11}\frac{\partial^2w_0}{\partial\theta^2}-E_{11}\frac{\partial\Phi}{\partial\theta}]\delta w_0\}d\theta dt=0\end{aligned}\tag{17}$$

Since δu_0, δw_0 and $\delta\Phi$ are independent, their coefficients must be zero. So:

$$\begin{aligned}&RI_2\ddot{\Phi}-\frac{1}{R}A_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}A_{11}\frac{\partial w_0}{\partial\theta}-\\ &\frac{1}{R}B_{11}\frac{\partial^2\Phi}{\partial\theta^2}-\frac{1}{R}E_{11}\frac{\partial w_0}{\partial\theta}-E_{11}\Phi=0\end{aligned}\tag{18}$$

$$\begin{aligned}&RI_1\ddot{w}_0+R[MV^2w''+2VM\dot{w}'+\\ &M\ddot{w}]\delta_d\left(R\theta-Vt\right)+(\frac{B_{11}}{R}-E_{11})\frac{\partial\Phi}{\partial\theta}+\\ &\frac{1}{R}A_{11}w_0-\frac{1}{R}E_{11}\frac{\partial^2w_0}{\partial\theta^2}=RMg\,\delta_d\left(R\theta-Vt\right)\end{aligned}\tag{19}$$

$$\begin{aligned}&RI_3\ddot{\Phi}-\frac{1}{R}B_{11}\frac{\partial^2u_0}{\partial\theta^2}-\frac{1}{R}B_{11}\frac{\partial w_0}{\partial\theta}-\\ &\frac{1}{R}D_{11}\frac{\partial^2\Phi}{\partial\theta^2}+E_{11}\frac{\partial w_0}{\partial\theta}+RE_{11}\Phi=0\end{aligned}\tag{20}$$

As the axis $z=0$ is located at the neutral axis of the beam cross section, so, $I_2=0$. Therefore, Eq. (18) gives:

$$\frac{\partial^2u_0}{\partial\theta^2}=-\frac{\partial w_0}{\partial\theta}-\frac{B_{11}}{A_{11}}\frac{\partial^2\Phi}{\partial\theta^2}-\frac{E_{11}}{A_{11}}\frac{\partial w_0}{\partial\theta}-\frac{RE_{11}}{A_{11}}\Phi \tag{21}$$

Combining Eqs. (20) and (21) gives:

$$\begin{aligned}&RI_3\ddot{\Phi}-\frac{1}{R}\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\frac{\partial^2\Phi}{\partial\theta^2}+E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{\partial w_0}{\partial\theta}+\\ &E_{11}\left(\frac{B_{11}}{A_{11}}+R\right)\Phi=0\end{aligned}\tag{22}$$

Eqs. (19) and (22) are the governing dynamic bending equations of the curved composite sandwich beam.

Also, the associated boundary conditions are:

$$w\left(0,t\right)=w''\left(0,t\right)=0 \tag{23}$$

$$w\left(L,t\right)=w''\left(L,t\right)=0 \tag{24}$$

Using the modal analysis method, the displacements of the beam are assumed as [22]:

$$\begin{aligned}&\phi\left(\theta,t\right)=\sum_{n=1}^{N}\phi_n\left(t\right)\cos\left(\frac{n\pi R\theta}{L}\right)\\ &w\left(\theta,t\right)=\sum_{n=1}^{N}w_n\left(t\right)\sin\left(\frac{n\pi R\theta}{L}\right)\end{aligned}\tag{25}$$

In which N is the number of considered modes, L is the beam length and n is the considered mode number. Also, $\phi_n\left(t\right)$ and $w_n\left(t\right)$ are the generalized coordinates and $\sin\left(\frac{n\pi R\theta}{L}\right)$

and $\cos\left(\frac{n\pi R\theta}{L}\right)$ are the normal modes corresponding to the simply supported boundary conditions.

Substituting Eqs. (25) into Eqs. (19) and (22), multiplying equation (19) by $\sin\left(\frac{i\pi R\theta}{L}\right)$ $(i \neq n)$, integrating along the beam length and using the orthogonality of the normal modes, gives ordinary differential equations in terms of time as:

$$\sum_{n=1}^{N}\left(I_3\ddot{\phi}_n+\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{n\pi}{L}\right)^2\right)\phi_n+ E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{n\pi}{L}w_n\right)=0 \quad (26)$$

$$\sum_{n=1}^{N}(I_1\ddot{w}_n+E_{11}(\frac{n\pi}{L})^2 w_n-(\frac{B_{11}}{R}-E_{11})(\frac{n\pi}{L})\phi_n +\frac{1}{R^2}A_{11}w_n)\delta_{ni}+\sum_{n=1}^{N}M[(\ddot{w}_n-V^2(\frac{Rn\pi}{L})^2 w_n]sin (\frac{n\pi Vt}{L})sin(\frac{i\pi Vt}{L})+M\sum_{n=1}^{N}2V\dot{w}_n(\frac{Rn\pi}{L})cos (\frac{n\pi Vt}{L})sin(\frac{i\pi Vt}{L})=Mgsin(\frac{i\pi Vt}{L}) \quad (27)$$

Using the operator method, the above two coupled equations are transformed to a differential equation as:

$$A_4(t)\frac{d^4\phi_n}{dt^4}+A_3(t)\frac{d^3\phi_n}{dt^3}+A_2(t)\frac{d^2\phi_n}{dt^2} +A_1(t)\frac{d\phi_n}{dt}+A_0(t)\varphi_n=A_5(t) \quad (28)$$

The coefficients of the above equation are presented in the appendix. The analytical solution of equation (28) with time dependent coefficients is impossible and the numerical method should be used. To do this, the fourth order Runge-Kutta method is employed. The initial conditions of the above differential equation are considered as:

$$\dddot{\phi}(0)=\ddot{\phi}(0)=\dot{\phi}(0)=\phi(0)=0 \quad (29)$$

3. Results

3.1. Validation of the Analysis

In this section, the present analytical model is validated through comparison of the present results with those of reference [23]. The material properties of the beam considered in different parts of the present research are presented in Table 1.

In the above Table, b is the width, H, the total thickness, E, the modulus of elasticity, G, the shear modulus, υ, Poisson's ratio and ρ, the density of the beam. Also, the superscripts f and c indicate the facesheet and the core respectively. In Fig. (2) the dynamic response results of the present research are compared with the results of reference [23] for the simply supported beam under a moving mass with M=1 kg and V=7.59 m/s at x=7L/16. As it is observed there is an excellent agreement with a maximum discrepancy of 4.13% between the results.

3.2. Parametric Study

In this section, the effects of different parameters such as the mass and the velocity of the moving mass, the radius of the curvature of the beam, the core thickness to the total thickness ratio and the stacking sequences of the face sheets on the dynamic response of the curved composite sandwich beam are investigated.

The dynamic deflection versus time curves of the beam at x=7L/16 for different values of M are illustrated in Fig. 3. As it can be seen, increasing the mass of the moving mass, results in the increase of the dynamic deflection. For example, at the time of 0.1383 s, when M is increased from 3 kg to 7 kg, the dynamic deflection increases by 108.01%.

The dynamic deflection versus time curves of the beam at x=7L/16 for different values of V are depicted in Fig. 4. It is observed that by increasing the velocity of the moving mass, the dynamic deflection decreases. This is likely to happen due to this fact that when the velocity of the moving mass increases, there is not enough time for occurrence of the beam deformation.

Table 1. Material properties of the considered beam

Width	b=0.2 (m)
Total thickness	h=0.4 (m)
Radius of curvature	R=3.5 (m)
Facesheet	$E_{11}^f = 39$ (GPa) $E_{22}^f = 8.66$ (GPa) $G_{12}^f = 3.8$ (GPa) $\upsilon_{12}^f = 0.28$ $\rho^f = 2100$ (Kg/m^3)
Core	$E_{11}^c = 3.74$ (GPa) $E_{22}^c = 0.172$ (GPa) $G_{12}^c = 0.202$ (GPa) $\upsilon_{12}^c = 0.229$ $\rho^c = 160$ (Kg/m^3)

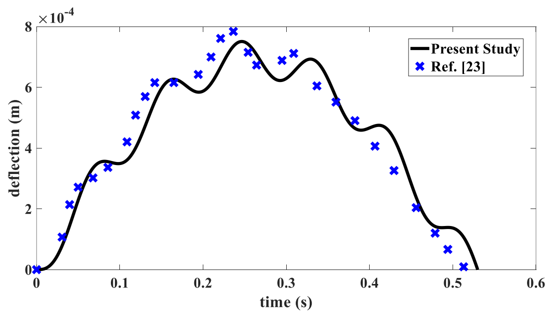

Fig. 2. The dynamic response of the simply supported curved composite sandwich beam under a moving mass at x=7L/16

For instance, at the time of 0.125 s, when V increases from 20 m/s to 30 m/s, the dynamic deflection decreases by 33.67%. However, the changes diminish for the higher values of V.

The dynamic deflection versus time curves of the beam at x=7L/16 for different values of R are presented in Fig. 5. It is seen that increasing the radius of the curvature, causes the dynamic deflection to be increased. The reason is likely due to this fact that when the radius of the curvature increases, the bending stiffness of the beam decreases. For example, at the time of 0.21 s, when R is increased from 3 m to 7 m, the dynamic deflection increases by 10%. However, the changes reduce rapidly for the higher values of R.

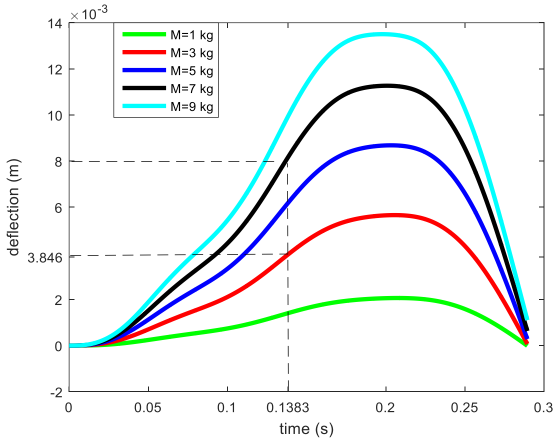

Fig. 3. The dynamic deflection versus time curves at x=7L/16 for different values of M

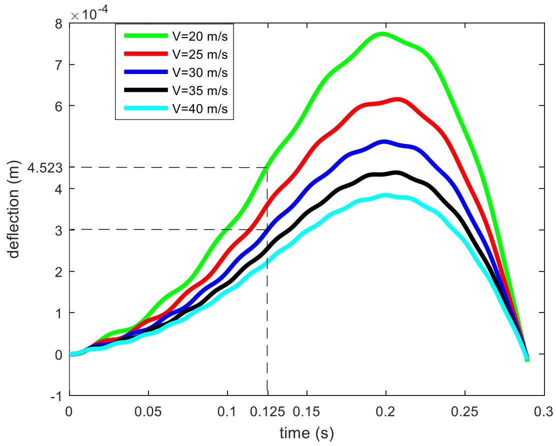

Fig. 4. The dynamic deflection versus time curves at x=7L/16 for different values of V

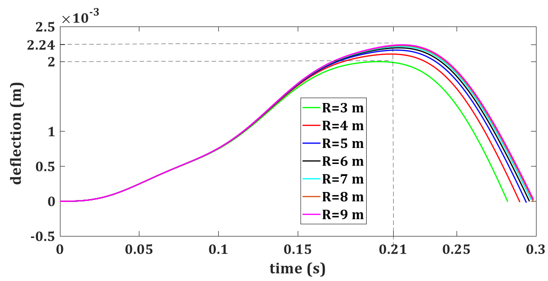

Fig. 5. The dynamic deflection versus time curves at x=7L/16 for different values of R

The dynamic deflection versus time curves of the beam at x=7L/16 for different values of the core thickness to the total thickness ratio of the beam hc/h are depicted in Fig. 6. It is observed that by increasing the value of hc/h, the dynamic deflection increases. For instance, at the time of 0.147 s, when hc/h increases from 0.3 to 0.7, the dynamic deflection increases by 124%. Also, the changes rise for the higher values of hc/h. This observation reveals that for a constant total thickness of the beam, increasing the face sheet thickness, improves the bending stiffness of the beam.

The dynamic deflection versus time curves of the beam at x=7L/16 for different stacking sequences of the face sheets are illustrated in Fig. 7. It is seen that the stacking sequence of the face sheet has a considerable effect on the dynamic deflection. For example, at the time of 0.17 s, when the stacking sequence differs from [-90/core/90] to [0/core/0], the dynamic deflection increases by 26.67%.

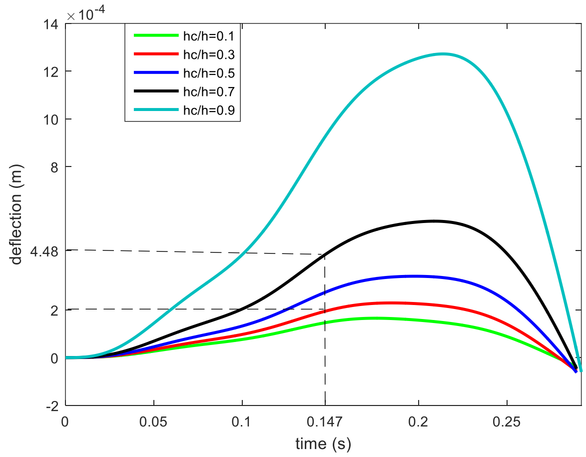

Fig. 6. The dynamic deflection versus time curves at x=7L/16 for different values of hc/h

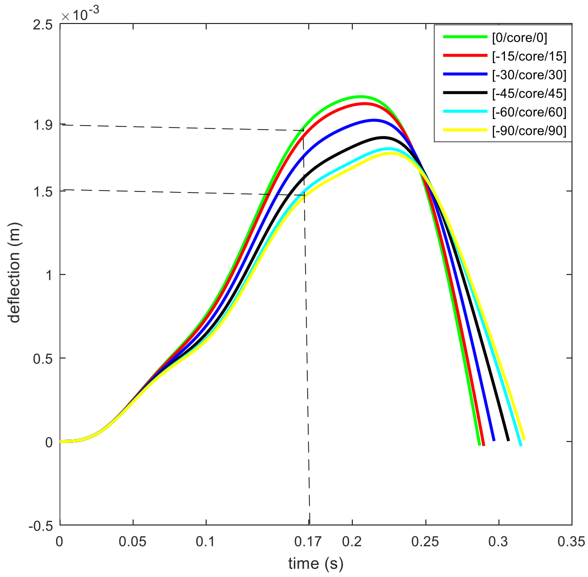

Fig. 7. The dynamic deflection versus time curves at x=7L/16 for different for different stacking sequences of the face sheets

4. Conclusions

In this paper the dynamic response of a simply- supported relatively thick composite sandwich curved beam under a moving mass with considering the rotary inertia and the transverse shear deformation was studied. In contrast to previous works, the geometry of the beam was considered to be in a curved form. Hamilton's principle was employed to derive the governing equations of the problem. After that, using the modal analysis method, the obtained partial differential equations were transformed to ordinary differential equations with time varying coefficients. Then, the ordinary differential equations were solved using fourth order Runge-Kutta method. The validity of the present analysis was confirmed through comparison of the present results and the results existed in the literature. Also, the effects of some important parameters such as the mass and the velocity of the moving mass, the radius of the curvature of the beam, the core thickness to the total thickness ratio and the stacking sequences of the face sheets on the dynamic response were investigated and discussed. The results of the present research can be important in design and application of the curved composite sandwich beams under moving mass.

Appendix

The coefficients of Eq. (28) are as the followings:

$$A_5(t) = -Mgsin\left(\frac{j\pi Vt}{L}\right)$$

$$A_4(t) = \sum_{N}^{i=1}\left(\begin{array}{c}\frac{I_1 I_3}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}}\delta_{ij} + \\ \frac{MI_3}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}}sin\left(\frac{i\pi Vt}{L}\right)sin\left(\frac{j\pi Vt}{L}\right)\end{array}\right)$$

$$A_3(t) = \sum_{i=1}^{N}\left(\begin{array}{c}\frac{2MVRI_3}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)}cos \\ \left(\frac{i\pi Vt}{L}\right)sin\left(\frac{j\pi Vt}{L}\right)\end{array}\right)$$

$$A_2(t) = \sum_{i=1}^{N}((\frac{I_1\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}} + \frac{I_3\frac{i\pi}{L}}{\left(\frac{B_{11}}{RA_{11}}+1\right)} + \frac{A_{11}I_3}{R^2E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}})\delta_{ij} +$$

$$(\frac{M\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}} - MV^2\left(\frac{Ri\pi}{L}\right)^2\frac{I_3}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}})sin\left(\frac{i\pi Vt}{L}\right)sin\left(\frac{j\pi Vt}{L}\right))$$

$$A_1(t) = \sum_{i=1}^{N}\left(2MVR\frac{\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)}\right)cos\left(\frac{i\pi Vt}{L}\right)sin\left(\frac{j\pi Vt}{L}\right)$$

$$A_0(t) = \sum_{i=1}^{N}((\frac{i\pi}{L}\frac{\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{\left(\frac{B_{11}}{RA_{11}}+1\right)} + \left(\frac{B_{11}}{R}-E_{11}\right)\left(\frac{i\pi}{L}\right) + \frac{\frac{1}{R^2}A_{11}\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)\frac{i\pi}{L}})\delta_{ij}$$

$$-MV^2R^2\frac{\left(E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\left(\frac{i\pi}{L}\right)^2\right)}{E_{11}\left(\frac{B_{11}}{RA_{11}}+1\right)}sin\left(\frac{i\pi Vt}{L}\right)sin\left(\frac{j\pi Vt}{L}\right))$$

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