Assessment of hydrostatic stress and thermo piezoelectricity in a laminated multilayered rotating hollow cylinder

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Abstract

In this paper, we built a mathematical model to study the influence of the initial stress on the propagation of waves in a hollow infinite multilayered composite cylinder. The elastic cylinder assumed to be made of inner and outer thermo piezoelectric layer bonded together with Linear Elastic Material with Voids (LEMV) layer. The model described by the equations of elasticity, the effect of the initial stress and the framework of linearized, three-dimensional theory of thermo elasticity. The displacement components obtained by founding the analytical solutions of the motion's equations. The frequency equations that include the interaction between the composite hollow cylinders are obtained by the perfect-slip boundary conditions using the Bessel function solutions. The numerical calculations carried out for the material PZT-5A and the computed non-dimensional frequency against various parameters are plotted as the dispersion curve by comparing LEMV with Carbon Fiber Reinforced Polymer (CFRP). From the graph it is clear that those are analyzed in the presence of hydrostatic stress is compression and tension.

1. Introduction

Composite materials are generally utilized in engineering structures because of their predominance over the basic materials in applications requiring high quality and solidness in lightweight parts. Thusly, the portrayal of their mechanical conduct is taking imperative part in basic plan. Procedures that incite transversely isotropic flexible properties in them make most cylindrical parts, for example, poles, wires, cylinders, funnels and strands. Displaying the proliferation of waves in these parts is significant in different applications, including ultrasonic nondestructive assessment systems, progression of room explore and numerous others. Smart materials are normally pre-stressed during the assembling procedure. As initial stresses are indivisible in surface acoustic wave gadgets and resonators, investigation of such impacts has been finished with various methodologies. A few creators have considered wave engendering in pre-stressed piezoelectric structure.


In this paper, we have built a mathematical model to study the effect of imposing thermal field on longitudinal wave propagation in a hollow multilayered composite cylinder under the influence of initial hydrostatic stress. The cylinder is made of tetragonal system material, such as PZT-5A. The displacement components are obtained by founding the analytical solutions of the motion’s equations. After applying suitable boundary conditions, the frequency equation is presented as a determinant with elements containing Bessel functions. The numerical computations obtained the roots of frequency equations. The dispersion curve carried for various parameters.

2. Formulation of the problem and basic equations

![Fig. 1. Geometry of the problem](image)

Longitudinal wave propagation in a homogeneous, transversely isotropic cylinder of tetragonal elastic material of inner and outer radius \( \tilde{r} \) and \( \tilde{R} \) and a subjected to an axial thermal and electric field is considered. The cylinder is treated as a perfect conductor and the regions inside and outside the elastic material is assumed to be vacuumed. The medium is assumed to be rotating with uniform angular velocity \( \Omega \). The displacement field, in cylindrical coordinates \((r, \theta, z)\) is given by

\[
\sigma^i_{rr} + \sigma^i_{rz} + r\gamma^i(\sigma^i_{rr}) + \rho(\tilde{\Omega} \times (\tilde{\Omega} \times \tilde{u})) \quad (1a)
\]

\[
+ 2(\tilde{\Omega} \times \tilde{u}_z) = \rho u_{tt}
\]
\[
\sigma_{r z}^i + \sigma_{zz}^i - r^{-1}\sigma_{rz}^i + \rho (\tilde{\nabla} \times (\nabla \times \tilde{u})) + 2(\tilde{\nabla} \times \tilde{u}_t) = \rho w_{tt} \quad (1b)
\]

The electric displacement equation
\[
\frac{1}{r} \frac{\partial}{\partial r} (r D_r^i) + \frac{\partial D_z^i}{\partial z} = 0 \quad (1c)
\]

The heat conduction equation
\[
K_1 (T_{rr}^i + r^{-1} T_{r}^i + r^{-2} T_{\theta}^i) + K_3 T_{zz}^i - \rho c_v T_{t}^i = \frac{T_0}{\partial \tau} [\beta_1 (e_{rr}^i + e_{r}^i) + \beta_3 (e_{zz}^i - p_3 \phi_z)] \quad (1e)
\]

The stress strain relations are given as follows
\[
\sigma_{rr}^i = c_{11} e_{rr}^i + c_{12} e_{r\theta}^i + c_{13} e_{zz}^i - \beta_1 T^i - e_{31} \phi_z^i
\]
\[
\sigma_{zz}^i = c_{13} e_{rr}^i + c_{12} e_{r\theta}^i + c_{33} e_{zz}^i - \beta_3 T^i - e_{33} \phi_z^i
\]
\[
\sigma_{r z}^i = c_{44} e_{r z}^i
\]
\[
D_r = e_{15} e_{r z}^i + e_{11} \phi_r^i
\]
\[
D_z = e_{31} (e_{rr}^i + e_{r\theta}^i) + e_{33} e_{zz}^i + e_{33} \phi_z^i + p_3 T
\]

The solutions of Eq. (4) is considered in the form
\[
u^i = U^i r \exp \{i(kz + pt)\}
\]
\[
w^i = \left(\frac{i}{k}\right) W^i \exp \{i(kz + pt)\}
\]
\[
\phi^i = \left(\frac{ic_{44}}{ae_{33}}\right) E^i \exp \{i(kz + pt)\}
\]
\[
\tau^i = \left(\frac{c_{44}}{p_3}\right) \left(\frac{r}{k}\right) E^i \exp \{i(kz + pt)\}
\]

Substitution of the Eqs.(3) and (2) into Eqs.(1) results in the following three-dimensional equation of motion, heat and electric conduction. We note that the first two equations under the influence hydrostatical stress become [31]:

\[
c_{11}(u_{rr}^i + r^{-1} u_{r}^i + r^{-2} u_{rr}^i) + c_{13} w_{r z}^i + c_{44}(u_{xx}^i + w_{xx}^i) + (e_{15} + e_{31}) \phi_{r z}^i + p_0 (u_{rr}^i + r^{-1} u_{r}^i + \beta_1 T^i) + \rho \Omega \left(\frac{-u + 2 \Omega w}{t}\right) = \rho u_{tt}^i
\]
\[
(c_{44} + c_{13})(u_{r z}^i + r^{-1} u_{z r}^i) + c_{33}(w_{r z}^i + w_{z r}^i) + \frac{\rho \Omega}{r} \left(\frac{-u + 2 \Omega w}{t}\right) = \rho u_{tt}^i
\]
\[
\frac{r^{-1} e_{15} (w_{r r}^i + u_{r z}^i) + e_{11} \phi_r^i + e_{33} u_{r z}^i + e_{33} \phi_z^i + \frac{p_3}{2} T = 0}{r^{-1} e_{15} (w_{r r}^i + u_{r z}^i) + e_{11} \phi_r^i + e_{33} u_{r z}^i + e_{33} \phi_z^i + \frac{p_3}{2} T = 0}
\]
\[
k_{11}(T_{rr}^i + r^{-1} T_{r}^i + r^{-2} T_{\theta}^i) + k_{33} T_{zz}^i - \frac{r_0 T_0}{\tau_0} d_{tt}^i = \left(\frac{e_{11}}{\rho_3}\right) \left(\frac{r}{k}\right) \left(\frac{r}{k}\right) E^i \exp \{i(kz + pt)\}
\]
\[
\begin{align*}
&[\varepsilon (1 + \bar{c}_{13})\nabla^2]U^i + [(1 - p_0)\varepsilon^2 - (\bar{c}_{33} - p_0)\varepsilon^2 + \zeta^2 + \chi^2 + 2]W^i + (\bar{e}_{15}\varepsilon^2 - \varepsilon^2)E^i - \\
&\varepsilon T^i = 0 \quad (6b)
\end{align*}
\]

\[
\varepsilon((\bar{d}_{31} + \bar{e}_{15})\varepsilon^2 U^i + (\bar{e}_{15}\varepsilon^2 + \varepsilon^2)W^i - \\
(\tilde{K}_{11}^2\varepsilon^2 + \tilde{K}_{33}\varepsilon^2)E^i - p\varepsilon T^i = 0 \quad (6c)
\]

\[
\bar{\beta}\nabla^2 U^i + \varepsilon W^i - p\varepsilon E^i + (i\tilde{K}_1\varepsilon^2 + i\tilde{K}_3\varepsilon^2 - \\
d)T^i = 0 \quad (6d)
\]

The above Eqs. [6] rearranged in the following form

\[
\begin{vmatrix}
\bar{c}_{11} - p_0\varepsilon^2 - s_1 + A_1 \\
A_5 \\
A_7 \\
A_9
\end{vmatrix}
\begin{vmatrix}
A_1 \\
A_5 \\
A_7 \\
A_9
\end{vmatrix}
= (\bar{U}_5, \bar{W}, \bar{E}, T) = 0
\]

Where

\[
A_1 = \frac{\zeta^2 + \chi^2 + 2}{2}, \quad A_2 = \frac{\varepsilon(1 + \bar{c}_{13})}{2}, \quad A_3 = \frac{\varepsilon(\bar{d}_{31} + \bar{e}_{15})}{2},
\]

\[
A_4 = -p_0\varepsilon^2, \quad A_6 = \varepsilon, \quad A_7 = \bar{K}_{11}^2\varepsilon^2 - \tilde{K}_{11}^2\varepsilon^2, \quad A_8 = -p_0\varepsilon^2
\]

\[
A_9 = i\tilde{K}_3\varepsilon^2 - \tilde{d}, \quad s_1 = (1 - p_0)\varepsilon^2, \quad s_2 = (\bar{c}_{33} - p_0)\varepsilon^2.
\]

Evaluating the determinant given in Eq. (7), we obtain a partial differential equation of the form:

\[
(\nabla^8 + \beta\nabla^4 + \gamma\nabla^6 + \delta\nabla^2 + \varepsilon) (U^i, W^i, E^i, T^i) = 0
\]

Factorizing the relation given in Eq. (8) into biquadratic equation for \((a^i_ja_j)^2\), i=1, 2, 3, 4 the solutions for the symmetric modes are obtained as

\[
U^i = \sum_{j=1}^{4}[A_jJ_n(\alpha_jx) + B_jN_n(\alpha_jx)],
\]

\[
W^i = \sum_{j=1}^{4}a^i_j[A_jJ_n(\alpha_jx) + B_jN_n(\alpha_jx)],
\]

\[
E^i = \sum_{j=1}^{4}b^i_j[A_jJ_n(\alpha_jx) + B_jN_n(\alpha_jx)],
\]

\[
T^i = \sum_{j=1}^{4}c^i_j[A_jJ_n(\alpha_jx) + B_jN_n(\alpha_jx)],
\]

Here \((a^i_jx) > 0\), for \((i = 1, 2, 3, 4)\) are the roots of algebraic equation

\[
(A^i_ja_j)^2 + B^i_j(a_ja_j)^2 + C^i_j(a_ja_j)^2 + \\
D^i_j(a_ja_j)^2 + E (U^i, W^i, E^i, T^i) = 0 \quad (10)
\]

The solutions corresponding to the root \((a, a)^2 = 0\) are not considered here, since \(J_n(0) = 0\), except \(n = 0\). The Bessel function \(J_n\) is used when the roots \((a, a)^2\), \((i = 1, 2, 3)\) are real or complex and the modified Bessel function \(I_n\) is used when the roots \((a, a)^2\), \((i = 1, 2, 3)\) are imaginary.

The constants \(a^i_j, b^i_j, c^i_j\) defined in Equation (10) can be calculated from the following equations

\[
[(\bar{d}_{31} - p_0)\varepsilon^2 - (1 - p_0)\varepsilon^2 + \zeta^2 + \chi^2 + 2] - e(1 + \bar{c}_{13})a^i_j - e(\bar{e}_{31} + \bar{e}_{15})b^i_j - \\
b^i_j = 0
\]

\[
\varepsilon(1 + \bar{c}_{13})\varepsilon^2 + [(1 - p_0)\varepsilon^2 - (\bar{c}_{33} - p_0)\varepsilon^2 + \zeta^2 + \chi^2 + 2]a^i_j + (\bar{e}_{15}\varepsilon^2 - \varepsilon^2)b^i_j - e^i_j = 0
\]

\[
\varepsilon((\bar{e}_{31} + \bar{e}_{15})\varepsilon^2 + (\bar{e}_{15}\varepsilon^2 + \varepsilon^2)a^i_j - \\
-K_{11}\varepsilon^2 + K_{33}\varepsilon^2)b^i_j - p\varepsilon c^i_j = 0
\]

\[
\bar{\beta}\nabla^2 + \varepsilon a^i_j - p\varepsilon b^i_j + (i\tilde{K}_1\varepsilon^2 + i\tilde{K}_3\varepsilon^2 - \tilde{d})c^i_j = 0
\]

3. Equation of Motion for Linear Elastic Materials with Voids LEMV

The displacement equations of motion and equation of equilibrium for an isotropic LEMV are

\[
(\lambda + 2\mu)(u_{rr} + r^{-1}u_r - r^{-2}u) + \mu u_{xx} \quad (11a)
\]

\[
+ (\lambda + \mu)w_{xx} + \beta E_x = \rho u_{tt}
\]

\[
(\lambda + \mu)(u_{r^2} + r^{-1}u_r) + \mu w_{rr} + r^{-1}w_{r} \quad (11b)
\]

\[
+ (\lambda + 2\mu)w_{xx} + \beta E_x = \rho w_{tt}
\]

\[
-\beta(u_r + r^{-1}u) - \beta w_x + \alpha E_{rr} + r^{-1}E_r + \\
\phi_{xx} - \delta k E_{tt} - \omega E_{tt} - \xi E = 0 \quad (11c)
\]

The stress in the LEMV core materials are

\[
\sigma_{rr} = (\lambda + 2\mu)u_r + \lambda r^{-1}u + \lambda w_x + \beta \phi
\]

\[
\sigma_{r} = \mu(u_t + w_r)
\]

The solution for Eq. (11) is taken as

\[
u = U_r\exp(kz + pt)
\]

\[
w = \frac{i}{R}W\exp(kz + pt)
\]

\[
(12)
\]
\[ E = \left( \frac{1}{r^2} \right) E \exp(i(kz + pt)) \]

The above solution in (11) and dimensionless variables \( x \) and \( \epsilon \), equation can be simplified as

\[
\begin{bmatrix}
(\lambda + 2\mu)\nabla^2 + M_1 & -M_2 & M_3 \\
M_2\nabla^2 & -M_2\nabla^2 + M_4 & M_5 \\
-\frac{\rho_1}{\rho} \nabla^2 + M_4 & M_5 & \alpha \nabla^2 + M_6
\end{bmatrix} \times \begin{bmatrix} u \\ w \\ E \end{bmatrix} = 0
\] \tag{13}

Where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} \)

\[ M_1 = \frac{\rho_1}{\rho} (ch)^2 - \frac{\mu}{\rho} \epsilon_x^2, \quad M_2 = (\lambda + \mu) \epsilon_x, \quad M_3 = \beta \epsilon, \quad M_4 = \frac{\rho_1}{\rho} (ch)^2 \frac{\partial}{\partial x} - (\lambda + \mu) \epsilon_x \epsilon_x, \quad M_5 = \beta \epsilon \]

\[ M_6 = \frac{\rho_1}{\rho} (ch)^2 \frac{\partial}{\partial x} - \alpha \epsilon_x^2 - i \omega (ch) - \xi \]

The Eq. (13) can specified as,

\[
(V^6 + PV^4 + QV^2 + R)(U, W, E) = 0
\] \tag{14}

Thus the solution of Eq.(14) is as follows,

\[
U = \sum_{j=1}^{3} [A_j J_0 (\alpha j, x) + B_j Y_0 (\alpha j, x)],
\]

\[
W = \sum_{j=1}^{3} a_j [A_j J_0 (\alpha j, x) + B_j Y_0 (\alpha j, x)],
\]

\[
E = \sum_{j=1}^{3} b_j [A_j J_0 (\alpha j, x) + B_j Y_0 (\alpha j, x)],
\]

\((\alpha j, x)^2\) are the roots of the equation when replacing \( V^2 = - (\alpha j, x)^2 \). The arbitrary constant \( a_j \) and \( b_j \) are obtained from

\[
M_2 V^2 + (\bar{\mu} V^2 + M_4) a_j + M_5 b_j = 0,
\]

\[
- M_3 V^2 + M_5 a_j + (\alpha V^2 + M_6) b_j = 0
\]

By taking the void volume fraction \( E = 0 \), and the lame’s constants as \( \lambda = \tilde{c}_{12}, \mu = \frac{\tilde{c}_{11} - \tilde{c}_{12}}{2} \) in the Eq. (11) we got the governing equation for CFRP core material.

4. Boundary conditions and frequency equations

The frequency equations can obtain for the following boundary condition

- On the traction free inner and outer surface
  \( \sigma_{rr} = \sigma_{r\theta} = E^t = T^t = 0 \) with \( l = 1, 3 \)
- At the interface \( \sigma_{rr} = \sigma_{r\theta} = \sigma_{r\phi} = E^i = T^i = 0, T^i = 0, D^i = 0 \)

Substituting the above boundary condition, we obtained as a 22x22 determinant equation

\[
|Y_{ij}| = 0, \quad (i, j = 1, 2, 3, \ldots, 22)
\] \tag{15}

At \( x = x_0 \) Where \( j = 1, 2, 3, 4 \)

\[ Y_{1j} = 2 \tilde{\epsilon}_{06} (\frac{\alpha^j}{x_0}) J_1 (\alpha^j, x_0) \]

\[ - \left[ (\alpha^j a^j) \tilde{c}_{11} + \tilde{c}_{13} a^j \tilde{c}_{31} + \tilde{c}_{33} \tilde{c}_{11} b^j \right] J_0 (\alpha^j a x_0) \]

\[ + \beta c^j J_1 (\alpha^j a x_0) \]

\[ Y_{2j} = \left( \tilde{c} + a^j + \tilde{c}_{15} b^j \right) (a^j) J_1 (\alpha^j, x_0) \]

\[ Y_{3j} = b^j J_0 (\alpha^j, x_0) \]

\[ Y_{4j} = \frac{h^j}{x_0} J_0 (\alpha^j, x_0) - (\alpha^j) J_1 (\alpha^j, x_0) \]

In addition, the other nonzero elements \( Y_{1,4+4}, Y_{2,4+4}, Y_{3,4+4} \) and \( Y_{4,4+4} \) are obtained by replacing \( J_0 \) by \( J_1 \) and \( J_0 \) by \( Y_1 \).

At \( x = x_1 \)

\[ Y_{5j} = 2 \tilde{\epsilon}_{06} (\frac{\alpha^j}{x_1}) J_1 (\alpha^j, x_1) \]

\[ - \left[ (\alpha^j a^j) \tilde{c}_{11} + \tilde{c}_{13} a^j \tilde{c}_{31} + \tilde{c}_{33} \tilde{c}_{11} b^j \right] J_0 (\alpha^j a x_1) \]

\[ \frac{h^j}{x_1} J_0 (\alpha^j, x_1) \]

\[ Y_{6j} = \left( \tilde{c} + a^j + \tilde{c}_{15} b^j \right) (a^j) J_1 (\alpha^j, a x_1) \]

\[ Y_{7j} = (\alpha^j) J_1 (\alpha^j x_1) \]

\[ Y_{8j} = - \tilde{\mu} (\tilde{c} + a^j) (a^j) J_1 (\alpha^j x_1) \]

\[ Y_{9j} = (\alpha^j) J_1 (\alpha^j x_1) \]
\[ \begin{align*}
Y_{7,j+n} &= -\alpha_j j_1(\alpha_j x_1) \\
Y_{6,j} &= \alpha_j j_0(\alpha_j x_1) \\
Y_{6,j+n} &= -\alpha_j j_0(\alpha_j x_1) \\
Y_{6,j} &= \beta_j j_0(\alpha_j x_1) \\
Y_{10,j} &= e_j(\alpha_j) j_1(\alpha_j x_1) \\
Y_{11,j} &= \frac{e_j}{X_1} j_0(\alpha_j x_1) - (\alpha_j) j_1(\alpha_j x_1)
\end{align*} \]

and the other nonzero element at the interfaces \( x = x_1 \) can be obtained on replacing \( j_0 \) by \( j_1 \) and \( j_0 \) by \( Y_i \) in the above elements. They are \( Y_{1,j+4}, Y_{1,j+n}, Y_{1,j+11}, Y_{1,j+14}; (i = 5, 6, 7, 8) \) and \( Y_{6,j+4}, Y_{6,j+n}, Y_{6,j+11}, Y_{6,j+14} \). At the interface \( x = x_2 \), nonzero elements along the following rows \( Y_{1,j}, (i = 12, 13, \ldots, 18 \text{ and } j = 8, 9, \ldots, 20) \) are obtained on replacing \( x_1 \) by \( x_2 \) and superscript 1 by 2 in order. Similarly, at the outer surface \( x = x_3 \), the nonzero elements \( Y_{1,j}, (i = 19, 20, 21, 22 \text{ and } j = 14, 15, \ldots, 22) \) can be had from the nonzero elements of first four rows by assigning \( x_3 \) for \( x_0 \) and superscript 2 for 1. In the case of without voids in the interface region, the frequency equation obtained by taking \( E = 0 \) in Eq. (11) which reduces to a \( 20 \times 20 \) determinant equation. The frequency equations derived above are valid for different inner and outer materials of 6mm class and arbitrary thickness of layers.

5. Numerical results and discussion

The frequency equation given in Eq. (15) is transcendental in nature with unknown frequency and wave number. The solutions of the frequency equations obtained numerically by fixing the wave number. The material chosen for the numerical calculation is PZT-5A. The material properties of PZT-5A used for the numerical calculation given below:

\[ \begin{align*}
C_{11} &= 13.9 \times 10^{10} \text{ Nm}^{-2}; \quad C_{12} = 7.78 \times 10^{10} \text{ Nm}^{-2}; \\
C_{13} &= 7.43 \times 10^{10} \text{ Nm}^{-2}; \quad C_{33} = 11.5 \times 10^{10} \text{ Nm}^{-2}; \\
C_{44} &= 2.56 \times 10^{10} \text{ Nm}^{-2}; \quad C_{56} = 3.06 \times 10^{10} \text{ Nm}^{-2}; \\
\beta_1 &= 1.52 \times 10^{9} \text{ NK}^{-1} \text{m}^{-2}; \quad T_0 = 298K; \\
\beta_2 &= 1.53 \times 10^{9} \text{ NK}^{-1} \text{m}^{-2}; \quad c_p = 4200 \text{ kg}^{-1} \text{K}^{-1}; \\
p_1 &= -452 \times 10^{-6} \text{ CK}^{-1} \text{m}^{-2}; \quad e_{33} = -6.98 \text{ C} \text{m}^{-2}; \\
K_i &= K_0 = 1.5 \text{ Wm}^{-1} \text{K}^{-1}; \quad e_{33} = 13.8 \text{ C} \text{m}^{-2}; \\
e_{15} &= 13.4 \text{ C} \text{m}^{-2}; \quad \rho = 7750 \text{ Kg} \text{m}^{-2}; \\
e_{11} &= 60.0 \times 10^{-10} \text{ C} \text{N}^{-1} \text{m}^{-2}; \\
e_{33} &= 5.47 \times 10^{-10} \text{ C} \text{N}^{-1} \text{m}^{-2}.
\end{align*} \]

The non-dimensional frequency versus the wave number and thickness \( h \) are plotted in Figures 2–5, which delineate the impacts of the underlying hydrostatic stress on the longitudinal vibrations of a hollow circular cylinder for the benefit of turning parameter \( \Omega \). From the Fig 2 and 3 there are a type of comparative changes gathered in non dimensional frequency against the wave number which differing in
rotational speed $\Omega$. whereas, the non-dimensional frequency expanding at the same time for the higher estimations of wave number to arrive at as far as possible and again diminished in Fig:1. From the Fig 4 and 5, there is a comparative change, which can be watched above all in non-dimensional frequency against the thickness which shifting in rotational speed $\Omega$. The study represents the conduct of turning at first focused on hollow cylinder. Notwithstanding the idea of non-dimensional frequency against hydrostatic pressure ($p_0 = -5, 0, 5 \times 10^6$) is watched.

Fig. 6. Variation of frequency vs. wave number against hydrostatic stress $p_0$ with $E = 0$

Fig. 7. Variation of frequency vs wave number against hydrostatic stress $p_0$ with $E = 0.5$.

Fig. 8. Variation of frequency versus Thickness against hydrostatic stress $p_0$ with $E = 0$.

The non-dimensional frequency versus the wave number and thickness $h$ is plotted in Figures 6–9, which represent the impacts of the underlying hydrostatic stress on the longitudinal vibrations of a hollow circular cylinder for the estimation of Electric parameter $E$. From the Fig 6 and 7 there is a slight change happened in non-dimensional frequency against the wave number which shifting in Electric parameter $E$ and the equivalent, which shows in hydrostatic pressure. Despite the fact that, the non-dimensional frequency which stays consistent by expanding in wave number, From the Fig 8 and 9 it is seen that the essential piece of non-dimensional frequency against the thickness which variety in Electric parameter $E$ and furthermore a similar which shows in hydrostatic pressure. From this, it is seen that the conduct of electric parameter is focused on at hollow cylinder. The idea of non-dimensional frequency against hydrostatic pressure ($p_0 = -5, 0, 5 \times 10^6$) consequently watched.

Fig. 9. Variation of frequency versus Thickness against hydrostatic stress $p_0$ with $E = 0.5$.

Fig. 10. Variation of nondimensional frequency versus thickness of cylinder with LEMV Layer and without hydrostatic stress.
The non-dimensional frequency versus the thickness $h$ against the with and without hydrostatic stress are plotted in 3D Figures 10–13, which illustrate the effects of the initial hydrostatic stress on the longitudinal vibrations of a LEMV and CFRP layers of the hollow circular cylinder. From the Fig 10 and 11 compared that the LEMV and CFRP layers how to vary without hydrostatic stress for increasing in thickness of the cylinder. From the Fig 12 and 13 compared that the LEMV and CFRP layers how to vary with hydrostatic stress ($p_0 = 5 \times 10^6$) for increasing of thickness of the cylinder. In both cases a linear nature observed in LEMV layers against the influences of with and without hydrostatic stress, but small deviations noted in CFRP layers against the influences of with and without hydrostatic stress.

The 3D Fig:14 -15 shows the non-dimensional strain against the wave number and thickness $h$, which illustrate the effects of the initial hydrostatic stress on the longitudinal vibrations of a hollow circular cylinder for the value of thermal parameter $\beta$.

6. Conclusion
The frequency equation for free axisymmetric vibration of the hollow circular cylinder with initial hydrostatic stress as core material is derived using three-dimensional linear theory of elasticity. Three displacement potential functions introduced to uncouple the equations of motion, electric and heat conduction. The frequency equation of the system consisting of rotating piezothermoelastic cylinder developed under the assumption of thermally insulated and electrically shorted free boundary conditions at the surface of the cylinder. The analytical equations numerically studied through the MATLAB programming for axisymmetric modes of vibration for PZT-5A material. The Dispersion curves shows the variation of the various physical parameters of the hollow circular cylinders with a rotation speed, thermal and electrical impacts. The damping observed is not significant due to the rotating effect of the circular hollow cylinder and the presence hydrostatic stress. In addition, the damping observed significant due to the thermal and electric effect of the circular hollow cylinder. The scattering of the frequency in LEMV and CFRP layers are observed in the presence of hydrostatic stress. The methods used in the present article are applicable to a wide range of problems in elasticity with different bonding layers.

Reference


[28] Habibi M., Hashemabadi D and Safarpour H., 2019. Vibration analysis of a high-speed rotat-