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A Novel Hybrid Genetic Modified Colliding Bodies Optimization for Designing of Composite Laminates

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KEYWORDS

Composite materials,
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Colliding bodies optimization,
Discrete variable

ABSTRACT

This study presents a robust hybrid meta-heuristic optimization algorithm by merging Modified Colliding Bodies Optimization and Genetic Algorithm that is called GMCBO. One of the inabilities of Colliding Bodies Optimization (CBO) is collapsing into the trap of local minima and not finding global optima. In this paper, to rectify this weak point, at first, some modifications are accomplished on the CBO process and then by using the concept of the genetic algorithm able to enhance the convergence rate, establishing a balance between the feature exploration and exploitation processes, the increasing power of finding global optimal design and escaping of local optimal. For evaluating the performance of the proposed method, the optimal design of laminated composite materials has been considered. Compare the results of structural analysis with GMCBO and other optimization methods shows a high convergence rate and its ability to find the global optimal solution of the proposed algorithm for structural optimization problems.

1. Introduction

In recent years, various meta heuristic optimization methods have been applied to solve engineering optimization problems. In general, based on the type of design variables, there are two categories of optimization methods, including discrete and continuous variable methods. Recent studies have mostly focused on optimal design of engineering problems with continuous variables meanwhile the standard size of sections available in the market has discrete values. Therefore, engineers must select the material from a list with available discrete values. Solving discrete optimization problems is far more difficult than continuous problems [1-2]. Usually, researchers use mathematical methods such as rounding on continuous solutions in order to solve discrete optimization problems. These methods may cause problems such as violation of the problem constraints (to fall in the infeasible space). This disadvantage has caused researchers to solve engineering optimization problems by using meta-heuristic optimization algorithms with discrete variables.

Genetic Algorithm is one of the meta-heuristic optimization methods that has led to a big evolution in solving engineering optimization

problems [3]. The Genetic Algorithm (GA) was first proposed by Goldberg based on inspiration from the laws governing the nature and survival of animals [4]. In recent years, many optimization methods have been developed such as Imperialist Competitive Algorithm (ICA) by Atashpaz and Lucas [5], Ant Colony Optimization (ACO) by Dorigo [6], Numbers Cup Optimization (NCO) by Riyahi et al. [7], Harmony Search (HS) by Jim et al. [8], Charged System Search (CSS) by Kaveh and Talatahari [9], Water Cycle Algorithm (WCA) by Eskandar et al [10], PathFinder Algorithm (PFA) by Yapici and Cetinkaya [11], Mine Blast Algorithm (MBA) by Sadollah et al. [12], Color harmony algorithm (CHA) by Zaeimi and Ghoddosian [13], Time Evolutionary Optimization (TEO) by Sheikhi et al. [14, 15], Colliding Bodies Optimization (CBO) by Kaveh and Mahdavi [16] and others.

Each of the proposed optimization methods, already introduced, have certain specifications. If the strengths and weaknesses of each method can be detected, two or more optimization methods can be combined to strengthen the strengths and overcome their weaknesses. For this purpose, researchers have recently focused on hybrid optimization techniques. Kaveh and Talatahari have worked on combining optimization

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methods of particle swarm, ant colony and harmony search [17], as well as charged particles and particle swarm [18], Sheikhi and Goddossian have merged imperialist competitive algorithm and the ant colony [19, 20], Shen et al have incorporated the two methods of particle swarm and tabu search [21], Sadallah et al have combined water cycle with mine blast algorithm [22], Sheikhi et al have incorporated Dolphin echolocation and Ant colony [23].

One of the strengths of the genetic algorithm is the ability to escape from the local optimum, especially with regard to optimization problems with discrete variables [24]. On the other hand, inability to escape from the local optimal trap is one of the weaknesses of the optimization algorithm of colliding bodies optimization [25].

This paper presented a robust hybrid meta heuristic optimization method deals with the combination of two methods of genetic algorithm and modified colliding bodies optimization. Here, it is tried to use the strengths of the genetic method to solve the weaknesses of colliding bodies optimization in the analysis of optimization problems with discrete design variables. The excellence criterion is associated with the function of meta-heuristic algorithms for a general exploration of the three main factors of solving problems faster, solving bigger problems (problems with high number of design variables and constraints), and increasing the potency of finding a better optimum point (reaching the global optimal point) [26, 27].

By combination of the two methods mentioned, the research has raised the convergence rate of the method in addition to increasing the potency of the colliding bodies optimization in engineering problems with discrete variables.

In this paper, in order to evaluate the efficiency of the proposed method, designing composite laminated materials using discrete design variables is proposed to achieve the minimum weight and required mechanical properties. For this purpose, programming of the hybrid optimization method of GMCBO and the analysis of the composite laminated are provided in MATLAB software.

2. Colliding Bodies Optimization Algorithm

Colliding bodies optimization algorithm was introduced for the first time by Kaveh and Mahdavi for continuous variables [16]. This algorithm is inspired by engineering rules governing objects dynamic collision at various speeds [28]. In this section, moving two objects with different speeds and masses is considered in order to obtain the equations governing objects collisions. Given that the contact forces are equal

and opposite during the collision, the linear motion magnitude of the system remains stable, and the law of conservation of the linear motion magnitude can be written as Eq. (1).

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2 \quad (1)$$

In the above relation, respectively, m_1 and m_2 represent the mass of objects; v_1 and v_2 represent the speed of objects before collision; v'_1 and v'_2 represent speed of objects after the collision. To obtain the velocity of objects after the collision requires another equation. For this purpose, the concept of an extraction coefficient can be used that indicates the ability of objects to recover their speed after the collision. The coefficient of restitution is expressed as the ratio of the relative velocity after collision to before it as the relation (2).

$$e = \frac{v'_2 - v'_1}{v_1 - v_2} \quad (2)$$

In the above relations e represents the restitution coefficient. Considering the Eqs. (1) and (2) simultaneously, as well as having the initial velocities of the objects and the restitution coefficient, the velocity of the objects after the collision can be calculated.

$$v'_1 = \frac{(m_1 - em_2)v_1 + (m_2 + em_2)v_2}{m_1 + m_2} \quad (3)$$

$$v'_2 = \frac{(m_2 - em_1)v_2 + (m_1 + em_1)v_1}{m_1 + m_2} \quad (4)$$

Some energy will be lost during the collision process. According to the classic collision theory, if the value of the restitution coefficient is 1, it means the maximum ability of the two objects to recover the velocity after the collision. This is the condition for an elastic collision without energy loss. On the other hand, if the value of the restitution coefficient is zero, it shows a completely plastic collision. In this case, objects stick together after the collision and their energy dissipation is maximal. All collisions occur at the point between the two modes of elastic and fully plastic collisions, and so the magnitude of the restitution coefficient is between 0 and 1. Details of the relations governing objects collision are presented in [29].

Colliding bodies optimization algorithm is a multi-agent algorithm like other meta-heuristic optimization algorithms. In this algorithm, for each agent (object), according to its fitness value, a mass value is assigned using Eq. (5).

$$m_i = \frac{1}{\frac{fit(i)}{1}}, i = 1, 2, \dots, N \quad (5)$$

$$\sum_{k=1}^N \frac{1}{fit(k)}$$

In the above relation, m_i represents the mass value of the particle i ; N is the total number of objects; and $fit(k)$ (or $fit(i)$) indicates the value of the objective function for the object k (or i). In order to choose a pair of objects to collide with each other, the objects are sorted from the highest to the smallest value, depending on their mass, and then divided into two equal groups of fixed and moving objects. Moving objects collide with steady objects to improve their position and move toward a new position after the collision. The velocity of steady objects is zero before the collision. The velocity of any moving object before the collision is calculated according to the Eq. (6). The velocity of any fixed and moving object after the collision is calculated as Eqs. (7) and (8), respectively.

$$v_{Mi} = X_{i-\frac{N}{2}} - X_i, \quad (6)$$

$$i = \frac{N}{2} + 1, \frac{N}{2} + 2, \dots, N$$

$$v'_{Si} = \frac{\left(m_{i+\frac{N}{2}} + em_{i+\frac{N}{2}} \right) v_{Si+\frac{N}{2}}}{m_i + m_{i+\frac{N}{2}}}, \quad (7)$$

$$i = 1, 2, \dots, \frac{N}{2}$$

$$v'_{Mi} = \frac{\left(m_i - em_{i-\frac{N}{2}} \right) v_{Mi}}{m_i + m_{i-\frac{N}{2}}}, \quad (8)$$

$$i = \frac{N}{2} + 1, \frac{N}{2} + 2, \dots, N$$

In Eqs. (6), (7) and (8), the M and S indexes are related to the characteristics of moving and fixed objects; x_i is the position of the object i ; v_i and v'_i , respectively, indicate the velocity of the i th object before and after the collision; and e is the restitution coefficient. In this paper, the restitution coefficient decreases in the process of optimization as the Eq. (9) [30].

$$e = 1 - \frac{iter}{iter_{max}} \quad (9)$$

In the above relation, $iter$ is the number of current repetitions and $iter_{max}$ represents the total number of repetitions during the optimization process. New positions of objects are updated according to their speed after the collision; as well as, the positions of fixed objects are updated. The new position of any fixed and moving object is calculated by Eqs. (10) and (11), respectively.

$$X_{Si}^{new} = X_{Si} + rand \cdot v'_{Si}, i = 1, 2, \dots, \frac{N}{2} \quad (10)$$

$$X_{Mi}^{new} = X_{Mi-\frac{N}{2}} + rand \cdot v'_{Mi}, \quad (11)$$

$$i = \frac{N}{2} + 1, \frac{N}{2} + 2, \dots, N$$

In the above relations, the $rand$ parameter is a random vector with uniform distribution in the range $[-1, 1]$. The sign " \odot " denotes the vectors element to element multiplication.

3. Hybrid Genetic Modified Colliding Bodies Optimization

In this section, a robust hybrid optimization algorithm is presented with combination of modified colliding bodies optimization and genetic algorithm (GMCBO) for discrete variables [31]. Genetic algorithm is one of the first meta-heuristic optimization algorithms containing various strengths to solve engineering problems, especially optimization problems with discrete variables. Here, first, a list of allowed discrete values of the design variables is given to the algorithm. Then, steps of the proposed algorithm are presented as the below [31].

Step 1- Random formation of the initial population in the feasible space.

Step 2- Assigning mass to objects based on the objective function with the Eq. (5).

Step 3. Elitism and storage of objects having the best position: At this stage, according to the concept of elitism in the genetic algorithm, between 5 to 10 percent of the elite population in each repetition are transmitted to the next stage alike. This step is in practice the use of the migration operator in the genetic algorithm. This operation can increase the efficiency and speed of the algorithm [9].

Step 4- To sort objects based on fitness and categorize them into two equal groups of fixed and moving objects.

Step 5. To assign the velocity of objects before the collision with the use of the Eq. (6) for moving objects and velocity zero for fixed objects.

Step 6. To assign velocity of objects after the collision using the Eq. (7) for moving objects and the Eq. 8 for fixed ones.

Step 7. To find new positions of the objects after the collision using the Eqs. (10) and (11) for fixed and moving objects, respectively.

Step 8. To update the position of each object based on a set of discrete variables.

Step 9. To check the non-violation of the constraints: At this stage, the status of the problem constraints is investigated. In the case of the constraint's violation, it is tried to move the object's position to the boundary of the possible space. For this purpose, the value of the second term on the right of the Eqs. (10) and (11) will be multiplied by a reducer coefficient.

Step 10. To apply the mutation operator used in the genetic algorithm: At this step, if the best solution obtained is not changed in several successive repetitions, in order to escape from the local minimum solutions, one of the design variables of the best solution is selected randomly and its amount changes.

Step 11. To apply the stop condition, the above steps will continue until the stop criterion is satisfied. In this article, reaching a predetermined number of repetitions is considered as the condition for stopping.

Figure 1 presents the flowchart of the hybrid genetic modified colliding bodies optimization algorithm.

4. Optimal Design of Composite Laminate

So far, researchers have considered various parameters such as weight, stiffness matrix [32, 33], natural frequency [34-37], etc. to the design of the optimal composite laminated materials. one of the most important aims of engineers in designing composite laminates is achieving mechanical properties that fulfill a particular purpose, which is a combination of the elements of the stiffness matrices A, B, and D [38]. In most problems, required values of these properties are set as goals for the designer. One can achieve the optimal design by suitably varying the number of layers and the thickness of them, arrangement, and orientation of the fibers within the layers.

Every layer of most composite materials available to designers has a specific thickness. Therefore, in case the layer thickness is considered as a design variable, the multilayer composite design becomes one of the optimization problems with discrete variables. On the other hand, if the orientation of each layer is considered as a design variable, the number of design variables becomes too large and it will become difficult to find a design that fulfills every constraint. Given the above circumstances, the present research has used the hybrid GMCBO optimization algorithm. In this section, the objective is to achieve the required effective mechanical properties and total weight for the laminated composite plate shown in Fig. 2.

The eight effective mechanical properties considered are two extensional stiffness elastic coefficients along two perpendicular directions (E_x and E_y), a shear modulus (G_{xy}), an effective major Poisson's ratio (ν_{xy}), two effective bending

stiffness moduli (E_x^f and E_y^f), an effective bending shear modulus (G_{xy}^f), and an effective bending Poisson's ratio (ν_{xy}^f). The studied objective function is a combination of the mentioned mechanical properties and the weight of the laminated composite plate, as shown in Eq. (12).

$$f(t_1, \theta_1, t_2, \theta_2, \dots, t_N, \theta_N) = \left(\sum_{i=1}^8 w_i \left(\frac{P_i^r - P_i^c}{P_i^r} \right)^2 \right) + w_9 \left(\frac{W^c - W^r}{W^r} \right)^2 \quad (12)$$

The design variables are the thicknesses of the various layers (t) and the orientation of the fibers in each layer (θ), which must be within the ranges presented in Eq. (13).

$$\begin{aligned} t_k^L &\leq t_k \leq t_k^U, k = 1, 2, \dots, N \\ \theta_k^L &\leq \theta_k \leq \theta_k^U, k = 1, 2, \dots, N \\ t^L &\leq t_i \leq t^U \end{aligned} \quad (13)$$

The coefficients w_1 to w_8 are the weighting factors of the eight effective mechanical properties of the composite material, and w_9 is the weighting factor of the weight of the laminate. The superscripts L and U represent the lower and upper bounds of the layer thickness and fiber orientation, respectively. Moreover, N and t_t are the number of layers and the total thickness of the laminate, respectively.

The parameters P^r are the objective values of the required eight effective mechanical properties, and P^c are the corresponding value obtained during the design process.

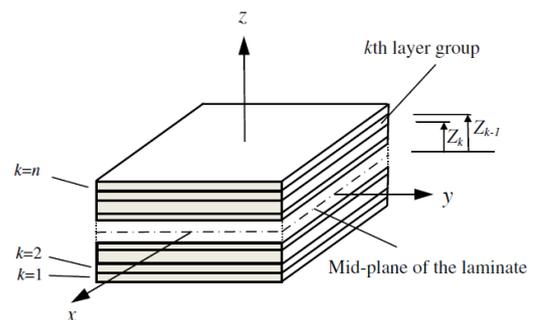


Fig. 2. Geometry and coordinate system of laminated composite plate [32]

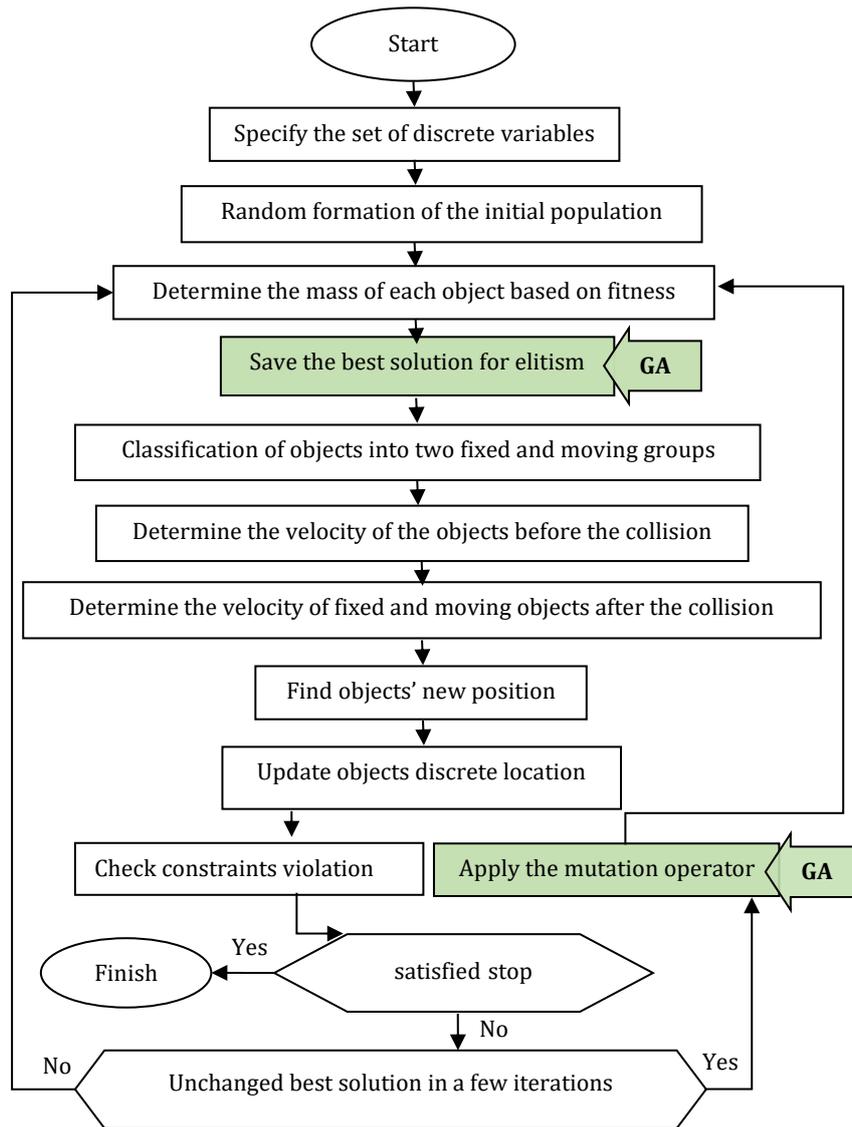


Fig. 1. the flowchart of GMCBO for discrete variables

Similarly, W^r and W^c are the objective and obtained weights of the laminated composite plate, respectively. The calculation of the mechanical properties P_i of composite laminates are defined in Eqs. (14) to (21).

$$E_x = \frac{|A|}{(A_{22}A_{66} - A_{26}^2)t_t} \quad (14)$$

$$E_y = \frac{|A|}{(A_{11}A_{66} - A_{16}^2)t_t} \quad (15)$$

$$G_{xy} = \frac{|A|}{(A_{11}A_{22} - A_{12}^2)t_t} \quad (16)$$

$$\nu_{xy} = \frac{A_{12}A_{66} - A_{16}A_{26}}{(A_{11}A_{22} - A_{12}^2)} \quad (17)$$

$$E_x^f = \frac{12|D|}{(D_{22}D_{66} - D_{26}^2)t_t^3} \quad (18)$$

$$E_y^f = \frac{12|D|}{(D_{11}D_{66} - D_{16}^2)t_t^3} \quad (19)$$

$$G_{xy}^f = \frac{12|D|}{(D_{11}D_{22} - D_{12}^2)t_t^3} \quad (20)$$

$$\nu_{xy}^f = \frac{D_{11}D_{66} - D_{16}D_{26}}{(D_{11}D_{22} - D_{12}^2)} \quad (21)$$

The number of design variables here is twice the number of layers. The design objective of this problem is to minimize the difference between the required objective values and the calculated values. Therefore, zero is the best objective function value. The required objective values are presented in Table 1.

Table 1. The required objective values for stiffness properties of the composite laminate.

$E_x = 92.136$ GPa	$E_y = 15.268$ GPa
$G_{xy} = 11.619$ GPa	$\nu_{xy} = 0.447$
$E_x^f = 106.951$ GPa	$E_y^f = 15.198$ GPa
$G_{xy}^f = 11.111$ GPa	$\nu_{xy}^f = 0.445$

The present research attempts to determine the thickness and fiber orientation for an eight-layer plate. The density of each base layer of the composite material used in the design is equal to $\rho = 0.056 \text{ kg/mm}^3$, and its engineering constants along the principal directions are $E_1 = 170 \text{ Gpa}$, $E_2 = 12 \text{ Gpa}$, $G_{12} = 4.5 \text{ Gpa}$ and $\nu_{12} = 0.3$.

The weighting factors used in Eq. 12 for the mechanical properties and the weight of the composite laminate are considered to be 0.1 and 0.2, respectively. Each base layer of the composite material is 0.13 mm thick; hence, the thickness of each designed layer can be a multiple of this value. The orientation angle of the fibers can vary between 0 and 180 degrees in 5-degree steps (the range of orientation angle of the fibers can be taken as -90 to 90). The use of discrete values for the design variables has led to a practical design and indicates the effectiveness of the proposed algorithm.

Figure 3a displays the convergence curve to the optimal design using the proposed algorithm. The variations in both the best optimal solution (bold blue curve) and the average of all solutions (red line curve) in each iteration have been plotted in this graph. The number of objects used in the algorithm is 20.

Figure 3b shows the variation of best optimal design at each iteration by SA. The optimal design is achieved after about 8500 iterations and has a lower convergence rate than GMCBO.

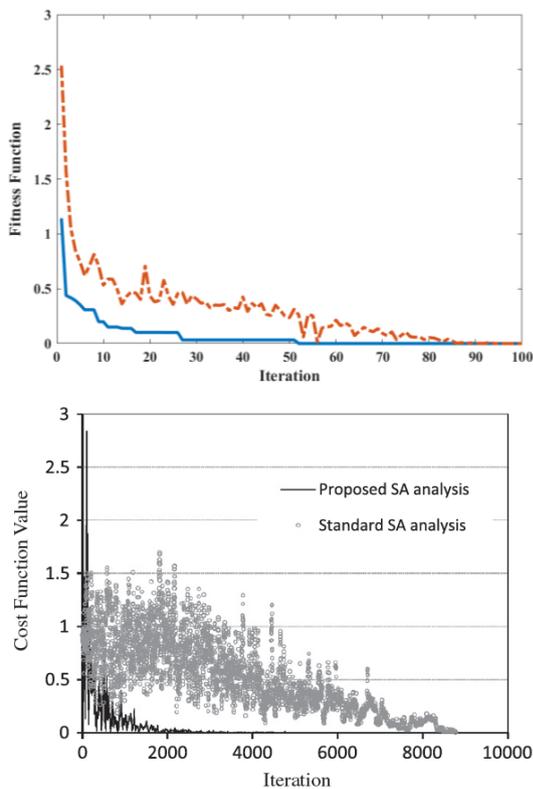


Fig. 3. (a) The best and average designs convergence curve for composite laminate using GMCBO (b) Variation of the best function using SA [33]

As seen in the convergence curve, the proposed algorithm has succeeded in achieving the optimal design using fewer than 60 iterations. The value of the objective function has become zero in the optimal point, indicating no difference between the required objective values and the corresponding calculated values. The total number of evaluations of the objective function is less than 1200 here. This shows the strength of the proposed algorithm in finding the optimal design for the laminated composite material.

The optimal layer thickness and fiber orientation are shown in Table 2. Given that the objective of the problem is minimizing the weight, the thickness of each layer has converged to its minimum value and the values of W^r and W^e have become equal. Furthermore, the required objective and calculated effective mechanical properties have become equal.

Figure 4 displays the changes in the movement of all objects during the convergence of the design toward the optimal point. The points are more dispersed during the initial iterations, but this dispersion becomes smaller with time and almost disappears after iteration number 90.

The number of evaluations of the objective function in the proposed method and SA are 1200 and 6879 respectively. GMCBO is about 83 percent fewer than that of simulated annealing (SA).

Figure 5 shows the plots of variations of relative error between the required objective and calculated values of mechanical properties during the design process using MGCBO. In all these plots, the difference in the values has reached zero after a number of iterations (maximum of 60 iterations).

In another case, the weighting factors for the mechanical properties and the weight of the composite laminate are considered equal to be 0.05 and 0.6, respectively. The GMCBO reached the previous results that proposed in Table 2. The reason for this is the feasibility of reaching a minimum weight, so the change in weight factor did not make a difference in the optimal point.

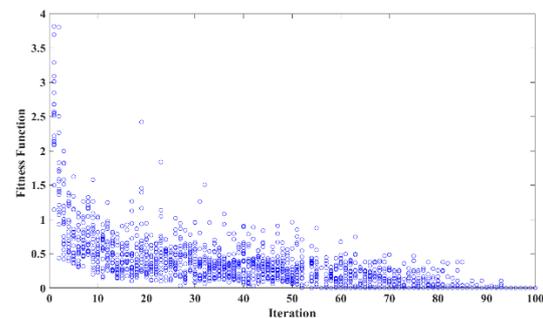


Fig. 4. The changes in the movement of all objects during the convergence of the design toward the optimal point

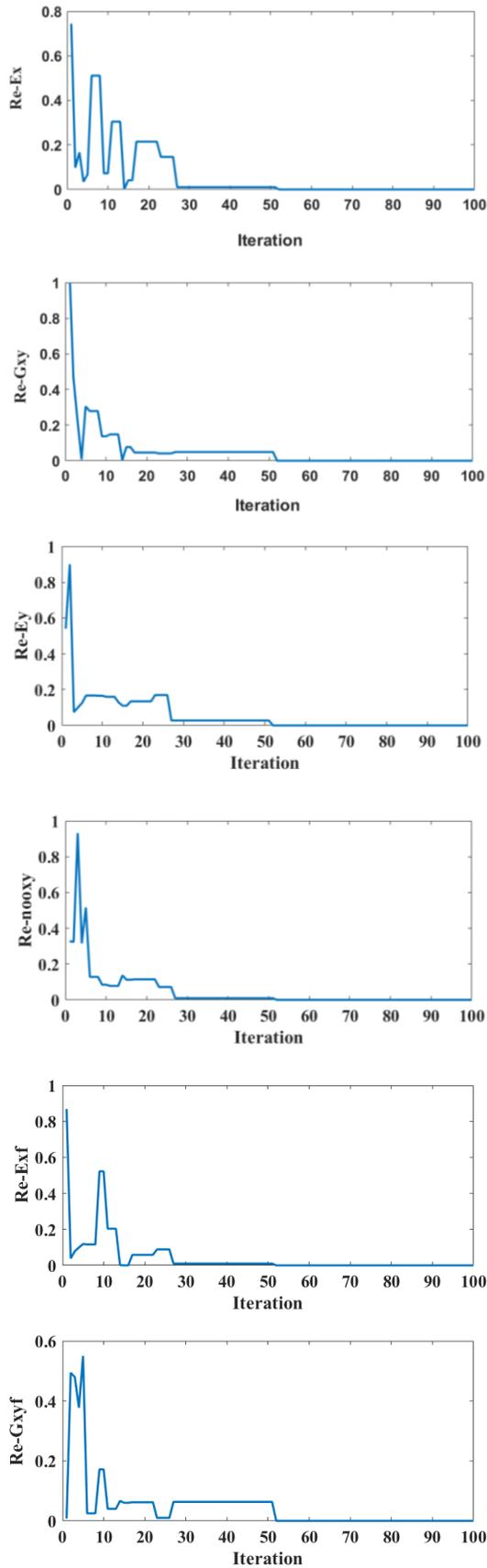
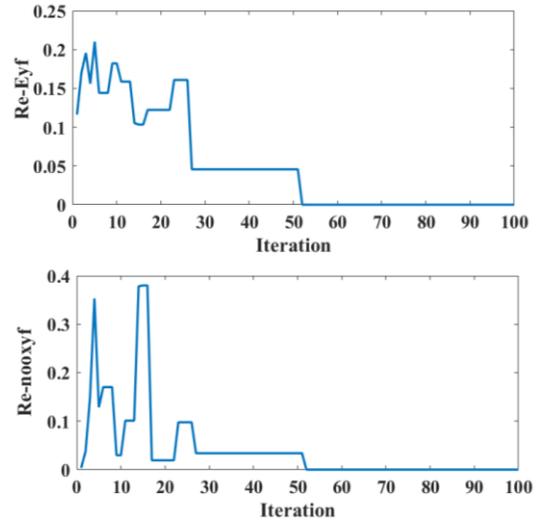


Fig. 5. The variations of relative error between the required objective and calculated values of mechanical properties during the design process using MGCBO



Continued Fig. 5.

Table 2. Optimal thickness and fiber orientation at layers

No. Layer	SA [32]		MGCBO	
	<i>t</i> (mm)	θ (deg)	<i>t</i> (mm)	θ (deg)
1	0.13	0.0	0.13	0.0
2	0.13	45	0.13	45
3	0.13	45	0.13	45
4	0.13	0.0	0.13	0.0
5	0.13	0.0	0.13	0.0
6	0.13	45	0.13	45
7	0.13	45	0.13	45
8	0.13	0.0	0.13	0.0

Since there are multi objective optimization problem, the differences in the values of mechanical properties and weight have increased in some cases during the optimization as expected; however, the overall reduction in difference is evident.

Figure 6 shows the variation in the relative error between the required objective and the calculated weight. Here, the objective weight is the minimum weight possible. This is achieved when all the plate layers have minimum thickness.

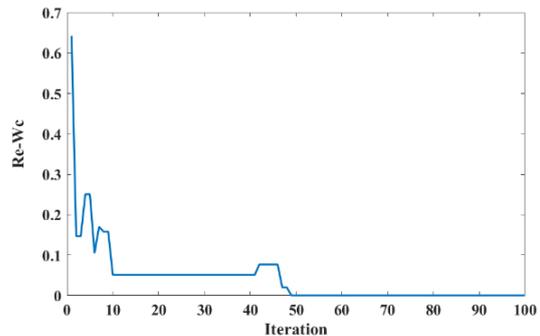


Fig. 6. The variation in the relative error between the required objective weight and the calculated weight during the design process using MGCBO

Table 3. Optimal thickness and fiber orientation for six layers

No. Layer	MGCBO	
	t (mm)	θ (deg)
1	0.13	0
2	0.26	45
3	0.13	180
4	0.13	180
5	0.26	45
6	0.13	0

Table 4. Statistical results for the optimization process in composite laminated by MGCBO

Standard Deviation	Worst	Average	Best
0.0414	0.1719	0.0403	0.0

In another case, the optimal design of composite laminate with six layers was done with the same objective functions as the previous target, and the result was proposed in Table 3. In this state, the values calculated and requested were converged to one value and so the performance of the proposed method has also been validated again.

The statistical results for the optimization process, namely the best, the average, and the worst values along with the standard deviation after 50 runs of the MGCBO are presented in Table 4.

5. Conclusions

Studying the behavior of composite materials is more challenging than common engineering materials. One reason for this is the large number of parameters affecting the behavior of the former materials. Therefore, effective tools are required to analyze and design a laminated composite material. For this purpose, the present research has proposed a hybrid optimization method involving a combination of a modified colliding bodies optimization algorithm and genetic algorithm. This method attempts to compensate for the weaknesses of CBO in discrete optimization problems using the strengths of GA. Combining these two methods has created a balance between the feature exploration and exploitation processes during optimization, avoided local optimum traps, and improved the convergence speed in addition to empowering the CBO method. A comparison of the results with previous works of research confirms the validity of the results. The zero-value obtained for the objective function indicates the absence of any difference between the required objective values and the calculated values. Hence, the algorithm has succeeded in achieving overall optimization.

References

- [1] Cheng, M.Y., Prayogo, D., Wua, Y.W., Lukito, M.M., 2016. A hybrid harmony search algorithm for discrete sizing optimization of truss structure. *Automation in Construction*, 69, pp. 21–33.
- [2] Lee, K.S., Geem, Z., Lee, W.S., Bae, K., 2005. The harmony search heuristic algorithm for discrete structural optimization. *Engineering Optimization*, 37(7), pp. 663–684.
- [3] Goldberg, D.E., 1989. *Genetic algorithms in search optimization and machine learning*. Boston: Addison-Wesley.
- [4] Ghoddosian, A., Sheikhi, M., 2013. *Meta heuristic optimization in engineering*. Semnan University Press.
- [5] Atashpaz-Gargari, E., Lucas C., 2007. Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition. *Proceedings of the IEEE Congress on Evolutionary Computation*, Singapore, September.
- [6] Dorigo, M., Maniezzo, V., Colorni, A., 1996. The ant system: optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics*, 26(1), pp. 29–41.
- [7] Riyahi Vezvari, M., Ghoddosian, A., Nikoobin, A., 2018. Numbers cup optimization: a new method for optimization problems. *Structural Engineering and Mechanics*, 66(4), pp. 465–476.
- [8] Geem, Z.W., Kim, J.H., Loganathan, G.V., 2001. A new heuristic optimization algorithm harmony search. *Simulation*, 76(2), pp. 60–68.
- [9] Kaveh, A., Talatahari, S., 2010. A novel heuristic optimization method: charged system search. *Acta Mechanica*, 213, pp. 267–86.
- [10] Eskandar, H., Sadollah, A., Bahreininejad, A., Hamdi, M., 2012. Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers and Structures*, 110, pp. 151–166.
- [11] Yapici, H., Cetinkaya, N., 2019. A new meta-heuristic optimizer: pathfinder algorithm. *Applied Soft Computing*, 78, pp. 545–568.
- [12] Sadollah, A., Bahreininejad, A., Eskandar, H., Hamdi, M., 2012. Mine blast algorithm for optimization of truss structures with discrete variables. *Computers and Structures*, 102, pp. 49–63.
- [13] Zaeimi, M., Ghoddosian, A., 2020. Color harmony algorithm: an art-inspired metaheuristic for mathematical function optimization, *Soft Computing*, 20, pp. 12027–12066.

- [14] Sheikhi, M., Delavar, M., Arjmand, M., 2016. Time evolutionary optimization: a new meta-heuristic optimization algorithm. *Proceedings of the 4th International Congress on Civil Engineering, Architecture and Urban Development*, Shahid Beheshti University, Tehran, Iran, December.
- [15] Sheikhi Azqandi, M., Delavar, M., Arjmand, M., 2020. An enhanced time evolutionary optimization for solving engineering design problems. *Engineering with Computers*, 36(2), pp. 763–781.
- [16] Kaveh, A., Mahdavi, V., 2014. Colliding bodies optimization method for optimum design of truss structures with continuous variables. *Advances in Engineering Software*, 70, pp.1–12.
- [17] Kaveh, A., Talatahari, S., 2009. Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures. *Computer and Structures*, 87, pp. 267–283.
- [18] Kaveh, A., Talatahari, S., 2012. A hybrid CSS and PSO algorithm for optimal design of structures. *Structural Engineering and Mechanics*, 42(6), pp.783–797.
- [19] Sheikhi, M., Ghoddosian, A., 2013. A hybrid imperialist competitive ant colony algorithm for optimum geometry design of frame structures. *Structural Engineering and Mechanics*, 46(3), pp. 403–416.
- [20] Sheikhi, M., Nooredin, N., Ghoddosian, A., 2018. Optimization of spring back in U-die bending process of sheet metal using ANN and ICA. *Structural Engineering and Mechanics*, 46(3), pp. 447–452.
- [21] Shen, Q., Shi, W.M., Kong, W., 2008. Hybrid particle swarm optimization and tabu search approach for selecting genes for tumor classification using gene expression data. *Computational Biology and Chemistry*, 32, pp. 53–60,
- [22] Sadollah, A., Eskandar, H., Bahreininejad, A., Kim, J.H., 2015. Water cycle, mine blast and improved mine blast algorithms for discrete sizing optimization of truss structures. *Computers and Structures*, 149, pp. 1–16,
- [23] Arjmand, M., Sheikhi Azqandi, M., Delavar, M., 2018. Hybrid improved dolphin echolocation and ant colony optimization for optimal discrete sizing of truss structures. *Journal of Rehabilitation in Civil Engineering*, 6(1), pp. 70–87.
- [24] Tsoulos, I.G., 2008. Modifications of real code genetic algorithm for global optimization. *Applied Mathematics and Computation*, 203, pp. 598–607.
- [25] Kaveh, A., Mahdavi, V., 2014. Colliding bodies optimization method for optimum design of truss structures with continuous variables. *Advances in Engineering Software*, 70, pp.1–12.
- [26] Das, S., Mukhopadhyay, A., Roy, A., Abraham, A., Panigrahi, B.K., 2011. Exploratory power of the harmony search algorithm: analysis and improvements for global numerical optimization. *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, 41(1), pp. 89–106.
- [27] Gandomi, A., Yang, X.S., Alavi, A., 2013. Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems. *Engineering with Computers*. 29(1), pp.17–35.
- [28] Kaveh, A., Mahdavi, V., 2015. *Colliding bodies optimization extensions and applications*. Springer.
- [29] Meriam, J.L., Kraige, L.G., 2006. *Engineering mechanics, dynamics*. John Wiley and Sons Publications.
- [30] Arjmand, M., Sheikhi, M., 2016. Optimal design of truss structure using colliding bodies optimization. *Proceedings of the 4th International Congress on Civil Engineering, Architecture and Urban Development*, Tehran, Iran, December 27–29.
- [31] Arjmand, M., Sheikhi Azqandi, M., 2018. A hybrid genetic modified colliding bodies optimization algorithm for design of structures with multi loads. *Iranian Journal of Marine Technology*, 5(2), pp. 14–25.
- [32] Javidrad, F., Nouri, R., 2011. A simulated annealing method for design of laminates with required stiffness properties. *Composite Structures*, 93, pp. 1127–1135.
- [33] Javidrad, F., Nouri, R., 2011. Stiffness optimization of laminates based on a modified parallel simulated annealing method. *Journal of Aeronautical Engineering*, 13(1), pp. 87–97.
- [34] Sadr, M.H., Ghashochi Bargh, H., 2012. Optimization of laminated composite plates for maximum fundamental frequency using Elitist-Genetic algorithm and finite strip method. *Journal of Global Optimization*, 54, pp. 707–728.
- [35] Ghashochi Bargh, H., Sadr, M.H., 2012. Stacking sequence optimization of composite plates for maximum fundamental frequency using particle swarm optimization algorithm. *Meccanica*, 47, pp. 719–730.
- [36] Ghashochi Bargh, H., Sadr, M.H., 2013, PSO algorithm for fundamental frequency optimization of fiber metal laminated panels, *Structural Engineering and Mechanics*. 47(5), pp. 713–727.

[37] Ghashochi Bargh, H., Sadr, M.H., 2017. Vibration Optimization of Fiber-Metal Laminated Composite Shallow Shell Panels Using an Adaptive PSO Algorithm, *Mechanics of Advanced Composite Structures*, 4(2), pp. 99-110.

[38] Hyer, M.W., 1998. *Stress analysis of fiber-reinforced composite materials*. McGraw Hill.