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Propagation Behavior of Love-Type Waves in a Poro-elastic Medium using Staggered Grid Finite Difference Scheme

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KEYWORDS

Dispersion curves
Phase velocity
Stability analysis

ABSTRACT

The analysis of the propagation behaviour of seismic waves in porous medium requires a mathematical backup. That is pictured here. The present study investigates the propagation behavior of Love-type waves in a poro-elastic medium. A staggered grid finite-difference (SFD) scheme in time-space domain formulation for Biot's equation of poro-elasticity is presented and applied to the problem. Complete theoretical seismograms for the horizontal and vertical components of displacement are obtained. The dispersion curves are evaluated considering the material parameters of different models. The stability analysis of the above numerical scheme is deliberated. From different graphs, it is noticed that there is a difference in phase velocity for various models. By observing the behaviour of the curve, we can understand the nature of the composite structure. Our outcomes also endorse that finite-difference modeling is an important numerical tool to acquire knowledge about the transmission of seismic waves in a porous medium. The agreement is excellent.

1. Introduction

We experience quite a few earthquakes every day throughout the world. Earthquakes can be violent enough to toss people around and destroy whole cities. Observations drive seismology, and improvements in instrumentation and data availability have often led to breakthroughs in seismology theory and our understanding of Earth structure. The information that seismology provides has widely varying degrees of uncertainty. The average radial seismic velocity structure of Earth has been known well, and the locations and seismic radiation patterns of earthquakes are now routinely mapped, but many important aspects of the physics of earthquakes themselves remain a mystery. Mathematics related to Earth systems faces major challenges by a new problem. It is also directly concerned with understanding the physical processes that cause earthquakes and seeking ways to reduce their destructive impacts on humanity. So, comprehensive research work can reduce the human losses and devastation of civil constructions. Therefore, these problems may be analyzed more accurately by modeling seismic wave propagation in a different medium.

Generally, waves are classified broadly into two groups – progressive waves and standing waves. Seismic waves may fall into either of these two categories. Progressive ones propagate away from sources, whereas standing ones generated by strong earthquakes represent vibrations of the earth and are also known as free oscillations of the Earth. Based on the spatial concentration of energy, spatial waves are categorized into surface waves and body waves. Body waves can propagate deep into the inner part of the corresponding medium, whereas surface waves concentrate along the surface of the medium. Body waves are faster than the surface waves as it is observed that they arrive even before the surface waves are emitted due to an earthquake. The higher velocity of the body wave correlates to its higher frequency than a surface wave. Whereas the surface waves generally have higher amplitudes with lengthier wavelengths than the body waves. They are also exclusively accountable for any mutilation and demolition accompanying the earthquakes. Surface waves,

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with changing amplitudes and gradually diminishing periods, seem to transmit through the earth's upper layer, between which the Love waves and Rayleigh waves are primarily significant originated through the earthquakes.

So far, quite a notable amount of research has been reported about the study of Love-type waves in several earthy materials. It has extensive applications in seismology, earthquake engineering, rock mechanics and geophysics, etc. Also, since the earth's layers are considered mostly as fluid-saturated porous layers, the analysis of seismic wave propagation within the complex poro-elastic media has noteworthy useful application in many areas such as soil dynamics, structure mechanics, petroleum engineering, survey techniques, geophysical prospecting, hydrology, oil exploration industry and other branches of applied sciences.

Elastic wave propagation in a poro-elastic medium is ruled by Biot's linear Theory [1-6]. This Theory is formed under these ideas:(a) the fluid phase is continuous so that detached pores are canned as a single matrix; (b) the porous media is isotropic, that is, the ratio of pore area and the solid occupied area is constant; (c) the minute pore size is much smaller than the seismic wavelength; (d) the distortions are small, which maintains the linearity of the mechanical processes;(e) the solid matrix is elastic. Poro-elasticity relates between the elastic deformation of a porous solid and the fluid flow and fluid gathering in the same material. Elastic wave propagation in complex poro-elastic media has wide application in geophysics and other branches of applied sciences, such as petroleum engineering, structure mechanics, and earth science. Comprehensive reviews of poro-elastic Theory have been described in [7-9] also. Most of the works related to the Love waves are well précised in various books, 'Wave propagation in Elastic Solid,' 'Elastic Waves in Layered Media' [10-11], etc., and numerous research papers [12-18], etc.

For modeling seismic waves transmission in porous reservoir rocks, numerical skill is an important tool for deducing seismic investigations in reservoir engineering. In modeling seismic waves that are purely elastic, numerical dispersion norms are well known and widely used, asserting that the smallest wavelength must be determined properly. The better the criteria are fulfilled, the better waveforms are well-maintained, and phase velocity are reproduced properly. Among the various numerical methods as finite element method, pseudo spectral method, finite difference method, we purely focused on the finite-difference scheme, well-known numerical method which is widely used in seismic modeling and migration. These are easy to implement and required comparatively less computation time and small memory, compared to other numerical techniques. For approximating temporal derivatives, a 2nd order finite-difference system is usually used to execute wave field recursion successfully and stably but has limits for the modeling accuracy. A smaller grid size or time step, may escalation the modeling accuracy but will require more calculation time. Staggered grid method is one among the many methods have been developed to improve the precision which do not rise the computation cost. Staggered grids (SG) is the most common stable finite difference scheme for solving the wave equation, where different material properties and variables are well-defined at different locations on the grid.

In numerical methods, the fundamental equations which define the problem, are solved including the boundary and initial conditions. Variables and constants are discretized in spatial and temporal derivatives on the regular or irregular grid, in which the equations are replaced by finite difference operators acting on variables at exact grid locations. Much information about the numerical modeling and propagation of seismic waves using the finite difference method is available in the following literature. The propagation of P-SV wave and SH wave in the heterogeneous media velocity-stress finite difference method has been used [19-20]. To improve the accuracy and stability of finite difference schemes, many authors have used and developed different types of finite difference schemes. Using the rotated staggered grid, the finite-difference modeling of viscoelastic and anisotropic wave propagation was conferred in [21-28], solved poro-elastic wave equations using staggered-grid finite-difference method. Wenzlau and Mueller [29] implement 2D SG and RSG methods to solve poro-elastic wave equations. Liu et al. [30-31] introduced a dispersion-relation time-space domain based on staggered-grid finite-difference schemes for modeling the scalar wave equation.

With increasing difficulties in environmental and engineering studies, modeling seismic wave propagation in the adjoining surface is vital and fundamental. Here we have investigated the propagation of Love-type waves in poro-elastic media considering proposed numerical methods and studied the stability conditions for the scheme, which is less attempted.

2. Biot's Equations of Dynamic Poro-elasticity

The Theory of wave and pore-pressure fields in porous media was developed by M.A. Biot [1-5]. Biot's equations for an isotropic fluid-saturated porous medium are given by

$$\rho\ddot{u} + \rho_f\ddot{w} = (\lambda_c + \mu)\nabla\nabla.u + \mu\nabla^2 u + \alpha M_f \nabla\nabla.w \quad (1)$$

$$\rho\ddot{u} + \rho_f\ddot{w} = (\lambda_c + \mu)\nabla\nabla.u + \mu\nabla^2 u + \alpha M_f \nabla\nabla.w \quad (2)$$

Using the velocity–stress formulation [20], the equations are derived from Biot (1 &2) for the poro-elastic medium. The pressure in a porous medium and the stress-strain relationship for a porous medium are stated as

$$(m_c\rho - \rho_f^2)\dot{U}_i = m_c\sigma_{ij,j} + \rho_f b q_i + \rho_f p_{,i} \quad (3)$$

$$(m_c\rho - \rho_f^2)\dot{q}_i = -\rho\sigma_{ij,j} - \rho b q_i - \rho p_{,i} \quad (4)$$

$$\dot{\sigma}_{ij} = \mu(U_{i,j} + U_{j,i}) + \delta_{ij}(\lambda_c U_{k,k} + \alpha M_f q_{k,k}) \quad (5)$$

$$\dot{p} = -\alpha M_f U_{k,k} - M_f q_{k,k} \quad (6)$$

Some material parameters with units and their inter dependence relation, required to express the porous medium are as follows (Table 1):

Table-1: Some material parameters required to express the porous medium:

Symbol, Name	Units	Related Equation (if any)
ρ_s Solid density	kg/m³	
ρ_f Fluid density	kg/m³	
ρ density	kg/m³	$\varphi\rho_f + (1-\varphi)\rho_s$
m Mass coupling coefficient	kg/m³	$\frac{T\rho_f}{\varphi}$
φ Porosity		
η fluid viscosity	Pa s	
k permeabilitym²		
T Tortuosity		
b The coefficient of friction	Pa s/m²	$\frac{\eta}{k}$
λ Lame constant	Pa	
λ_c Lame constant of saturated Medium	Pa	$\lambda_c = \lambda + \alpha^2 M$
μ Shear modulus	Pa	
M Fluid storage coefficient	Pa	$\left[\frac{\alpha-\varphi}{K_s} + \frac{\varphi}{K_f}\right]^{-1}$
α Biot coefficient of effective stress		$1 - \frac{K_d}{K_s}$
K_s Solid bulk modulus	Pa	
K_f Fluid bulk modulus	Pa	
K_d Drained bulk modulus	Pa	$\lambda + \frac{2}{3}\mu$

Here the stress is denoted by σ, p represent by pressure, v is solid velocity and the time derivative of u (solid displacement), and fluid velocity q, relative to the solid is the time derivative of w (fluid displacement relative to the solid).

Replacing the formulation of displacement-stress of equations (5) and (6) into the formulation of displacement-stress of equations (3) and (4), the equations then transformed to

$$(m\rho - \rho_f^2)D_{tt}u_i - mD_j[\mu(D_j u_i + D_i u_j) + \delta_{ij}(\lambda D_k u_k + \alpha M D_k w_k)] - \rho_f b D_t w_i + \rho_f D_i[\alpha M D_k u_k + M D_k w_k] = 0 \quad (7)$$

$$(m\rho - \rho_f^2)D_{tt}w_i + \rho_f D_j[\mu(D_j u_i + D_i u_j) + \delta_{ij}(\lambda D_k u_k + \alpha M D_k w_k)] + \rho b D_t w_i - \rho D_i[\alpha M D_k u_k + M D_k w_k] = 0 \quad (8)$$

3. Formulation of the Propagation of Love-type Waves.

Let us consider the coordinate system in such a way x-axis is parallel to the direction of wave propagation, and z-axis is vertically downward to the direction of wave propagation. The structure to be analyzed is shown in Fig. 1, in which we have considered different poro-elastic medium finite thickness h. The region $-h < z < 0$ is occupied by the poro-elastic medium.

The conventional conditions for Love-type waves propagation are.

i.e., $\vec{u}_1 = (0, u_y, 0)$and $\vec{w}_1 = (0, w_y, 0)$

where $u_y = u_y(x,z,t)$ *is the solid displacement* and $w_y = w_y(x,z,t)$ *is the fluid displacement component*.

The equations (7) and (8), along with the stress-strain relations, are reduced to the form

$$[(m\rho - \rho_f^2)D_{tt} + (-m\mu)D_{xx} + (-m\mu)D_{zz}]u_y - \rho_f b D_t w_y = 0 \quad (9)$$

$$[\rho_f \mu D_{xx} + \rho_f \mu D_{zz}]u_y + [(m\rho - \rho_f^2)D_{tt} + \rho b D_t]w_y = 0 \quad (10)$$

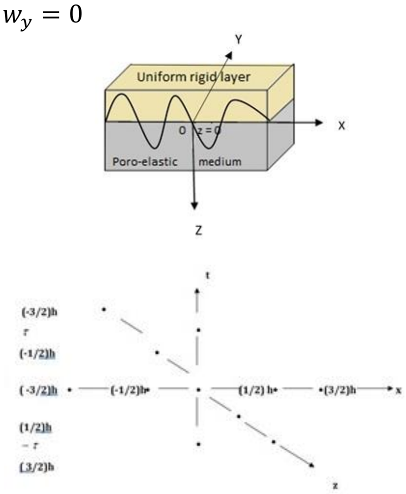

Fig.1. The structure of the problem

The equations (9) and (10) can be written as

$$\begin{bmatrix} (m\rho-\rho_f^2)D_{tt}+(-m\mu)D_{xx}+(-m\mu)D_{zz} & (-\rho_f b)D_t \\ \rho_f\,\mu D_{xx}+\rho_f\,\mu D_{zz} & (m\rho-\rho_f^2)D_{tt}+\rho b D_t \end{bmatrix} \begin{bmatrix} u_y \\ w_y \end{bmatrix}=0 \quad (11)$$

We have deliberated here the staggered grid finite difference method for spatial derivatives as

$$\left.\begin{aligned} D_x u &= \frac{\partial u}{\partial x} = \frac{1}{h}\{c_1\left[u\left(x+\tfrac{1}{2}h,z\right)-u\left(x-\tfrac{1}{2}h,z\right)\right] \\ &+c_2\left[u\left(x+\tfrac{3}{2}h,z\right)-u\left(x-\tfrac{3}{2}h,z\right)\right]\} \\ D_z u &= \frac{\partial u}{\partial z} = \frac{1}{h}\{c_1\left[u\left(x,z+\tfrac{1}{2}h\right)-u\left(x,z-\tfrac{1}{2}h\right)\right] \\ &+c_2\left[u\left(x,z+\tfrac{3}{2}h\right)-u\left(x,z-\tfrac{3}{2}h\right)\right]\} \end{aligned}\right] \quad (12)$$

According to the Von Neumann analysis, the solution of the plane-wave equation is taken as

$$u(x,z,t) = Ae^{i[k_x(x+mh)+k_z(z+jh)-\omega(t+n\tau)]}$$

where k_x, k_z are the wavenumbers and ω is the angular frequency we are getting,

$$D_{xx} = -\frac{4}{h^2}[\{c_1^2-2c_1c_2\left(1-4\cos^2\frac{k_xh}{2}\right)\}\sin^2\frac{k_xh}{2} + c_2^2\sin^2\frac{3k_xh}{2}] \quad (13)$$

$$D_{zz} = -\frac{4}{h^2}[\{c_1^2-2c_1c_2\left(1-4\cos^2\frac{k_zh}{2}\right)\}\sin^2\frac{k_zh}{2} + c_2^2\sin^2\frac{3k_zh}{2} \quad (14)$$

Generally, the higher-order finite-difference on the temporal derivative scheme requires large memory and is unstable; therefore, the second-order finite difference scheme is usually used for temporal derivatives, as stated below.

$$D_{tt}u = \frac{\partial^2 u}{\partial t^2} \approx \frac{1}{\tau^2}[-2u^t+u^{t+\tau}+u^{t-\tau}] \quad (15)$$

Further simplifying, using (12-14), from equations (11), we are getting the dispersion equation as

$$\delta = \frac{v_{FD}}{v} = \frac{1}{R\pi\frac{h}{\lambda}}\sin^{-1}\left(\frac{\tau}{h}\sqrt{\frac{m\mu A}{m\rho-\rho_f^2}}\right) \quad (16)$$

where $R = \frac{v\tau}{h}$, the courant number, $k_x = k\cos\theta$ and $k_z = k\sin\theta$, where θ, being the propagation angle of the plane wave, $v = \sqrt{\mu/\rho}$, the share wave velocity and

$$\begin{aligned} A &= (c_1^2-2c_1c_2)\left[\sin^2\left(\pi\frac{h}{\lambda}\cos\theta\right) + \sin^2\left(\pi\frac{h}{\lambda}\sin\theta\right)\right] \\ &+2c_1c_2\left[\sin^2(2\pi\frac{h}{\lambda}\cos\theta)+\sin^2(2\pi\frac{h}{\lambda}\sin\theta)\right] \\ &+c_2^2\left[\sin^2(3\pi\frac{h}{\lambda}\cos\theta)+\sin^2(3\pi\frac{h}{\lambda}\sin\theta)\right] \end{aligned}$$

There is no dispersion if δ is equal to 1, and a large dispersion will be arisen if δ is far from 1.

4. Stability Analysis for Love-type Waves

The recursion equation of finite difference scheme, obtained from equation (11), is

$$(m\rho-\rho_f^2)D_{tt} = m\mu D_{xx} + m\mu D_{zz}$$

can be written as follows:

$$\begin{aligned} \frac{(m\rho-\rho_f^2)}{\tau^2}&\left[-2u_{0,0}^0+u_{0,0}^1+u_{0,0}^{-1}\right] = m\mu \\ &\begin{bmatrix} (c_1^2+2c_1c_2)u_{1,0}^0+2(c_1^2+c_2^2)u_{0,0}^0+2c_1c_2u_{2,0}^0 \\ +(c_1^2+2c_1c_2)u_{-1,0}^0+2c_1c_2u_{-2,0}^0+c_2^2(u_{3,0}^0+u_{-3,0}^0) \end{bmatrix} \\ +m\mu &\begin{bmatrix} (c_1^2+2c_1c_2)u_{0,1}^0+2(c_1^2+c_2^2)u_{0,0}^0+2c_1c_2u_{0,2}^0 \\ +(c_1^2+2c_1c_2)u_{0,-1}^0+2c_1c_2u_{0,-2}^0+c_2^2(u_{0,3}^0+u_{0,-3}^0) \end{bmatrix} \end{aligned}$$

So we are getting from the above equation

$$\begin{aligned} u_{0,0}^1 &= [2+4R^2(c_1^2+c_2^2)]u_{0,0}^0 \\ &+R^2\begin{bmatrix} (c_1^2+2c_1c_2)\{(u_{1,0}^0+u_{-1,0}^0)+(u_{0,1}^0+u_{0,-1}^0)\} \\ +(2c_1c_2)\{(u_{2,0}^0+u_{-2,0}^0)+(u_{0,2}^0+u_{0,-2}^0)\} \\ +c_2^2\{(u_{3,0}^0+u_{-3,0}^0)+(u_{0,3}^0+u_{0,-3}^0)\} \end{bmatrix} \\ &-u_{0,0}^{-1} \end{aligned} \quad (17)$$

where $R = \frac{(m\rho-\rho_f^2)}{m\mu}$

Using the conventional Eigen-value method of stability analysis, let us consider:

$$\begin{bmatrix} p_{m,m}^0 = u_{m,m}^0;\ q_{m,m}^0 = u_{m,m}^{-1}; \\ U_{m,m}^0 = (p_{m,m}^0, q_{m,m}^0)^T = W^0e^{i(k_xmh+k_zmh)}; \\ U_{m,m}^1 = (p_{m,m}^1, q_{m,m}^1)^T = W^1e^{i(k_xmh+k_zmh)} \end{bmatrix} \quad (18)$$

Using equation (18) in equation (17), we obtain, according to Liu et al. (30), is

$$W^1 = GW^0 = \begin{bmatrix} g & -1 \\ 1 & 0 \end{bmatrix} W^0$$

where G is the transition matrix and

$$g = [2+4R^2(c_1^2+c_2^2)] + 2R^2\begin{bmatrix} (c_1^2+2c_1c_2)\{\cos(k_xh)+\cos(k_zh)\} \\ +(2c_1c_2)\{\cos(2k_xh)+\cos(2k_zh)\} \\ +c_2^2\{\cos(3k_xh)+\cos(3k_zh)\} \end{bmatrix} \quad (19)$$

The recursion relations of the finite difference scheme will be stable if the absolute values of the eigenvalues of the transition matrix are less than or equal to 1. The roots of the eigenvalue equation ${\lambda_1}^2 - g\lambda_1 + 1 = 0$ will be less than or equal to 1 if $|g| \leq 2$. Since the error usually increases with the increase of the wavenumber, let us consider the maximum wavenumber (Nyquist frequency) as

$$k_x = k_z = \frac{\pi}{h} \quad (20)$$

Using the equation (20) into the equation (19), we have $g = 2$

Therefore, a staggered grid scheme is always stable.

5. Numerical Calculations

Here we have considered four different models, mentioned in Table 2. Based on the dispersion equations (11), numerical results show the propagation characteristics of Love-type waves in different media, respectively. In all the figures, curves have been plotted as phase velocity $\frac{v_{Fd}}{v}$ along vertical axis against the grid points per wavelength h/λ_2 along the horizontal axis. The range of h/λ_2 is taken between the value of 0.02 and 0.4.

Figure 2 to 5 has been plotted to understand the propagation of Love-type wave velocity in the poro-elastic layered media.

Here we have chosen four models following [26].

We have observed how Love-type waves behave when are passing through poro-elastic media and presented them graphically.

Material constants for different media according to [26].

Table-2: Material constants for different media according to O'Brien (26)

	Model 1	Model 2	Model 3	Mode 4
h (m)	2.5	1e – 3	3.0	1.0
τ (s)	0.0005	1e – 7	3e – 4	1e – 4
ρ_f (kg/m^3)	1000	1000	1000	1000
ρ (kg/m^3)	2120	2120	2120	2050
m (kg/m^3)	6666.667	6666.667	3333.33	3333.33
b (Pas/m^2)	0	3333.33	10^9	$3.33\text{x}10^4$
λ (GPa)	3.5	4.5	4	3.5
μ (GPa)	3.5	4.5	3.0	3.5
M (GPa)	5.462	3.562	5.512	5.462
α	0.417	0.5	0.4	0.417

6. Discussions concerning Graphical Representations

Figures 2 to 5 display the dispersion curves of the propagation of Love-type waves concerning different grid points per wavelength at different values of time *t,* where *t* has been calculated with unconditionally stable time step τ_s.

From Fig. 2, Fig. 4, and Fig. 5, we have observed that when time is increasing, the phase velocity of Love-type waves is also increasing. In the case of Fig.2 and Fig. 4, velocity increases with time very marginally. It is also found that dispersion is more for the lower wavelength values and becomes less when we increase the wavelength.

Nevertheless, in the case of Fig. 3, velocity decreases with time. According to the dispersion curves, it is noticeable that velocity increases between 0.02 to 0.26 of wavelength at different time step velocity then decreases again at a different step.

We have plotted these graphs using the material parameters of different models (Table2). From these figures except Fig. 3, it can be illustrated that the dispersion curves are nearly consistent for different times.

So we can say model 1, model 3, and model 4 are stable.

From these curves, it is noticed that staggered grid methodologies deliver stable results.

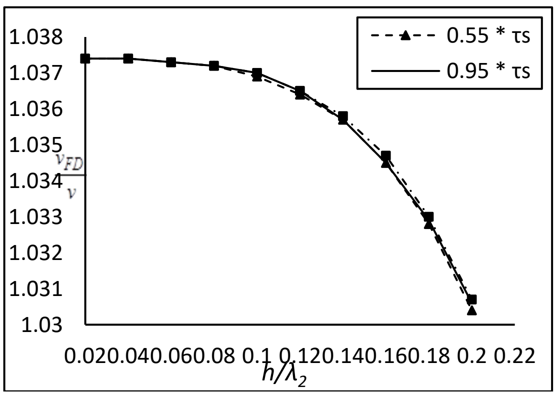

Fig. 2. Dispersion curves of Love-type waves at different time step using material parameters of Model 1

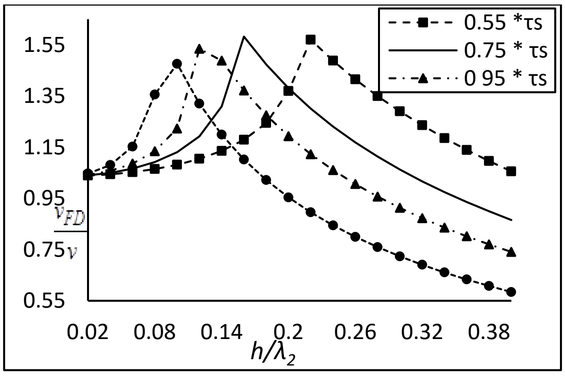

Fig. 3. Dispersion curves of Love-type waves at different time step using material parameters of Model 2

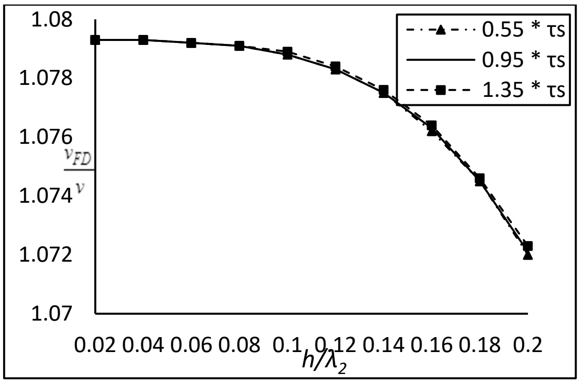

Fig. 4. Dispersion curves of Love-type waves at different time step using material parameters of Model 3

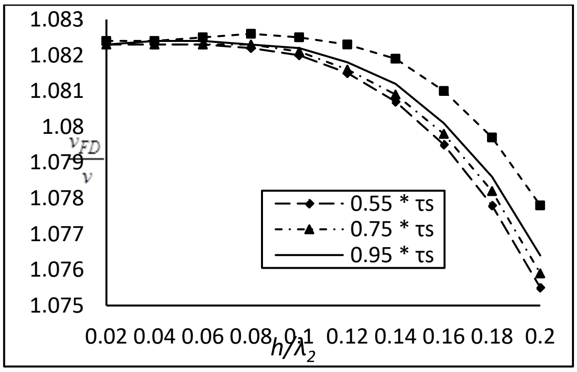

Fig. 5. Dispersion curves of Love-type waves at different time step using material parameters of Model 4

7. Conclusions

Seismic wave modeling is an effective way to understand the complexity of earthquakes, for which finite-difference methods are often applied. The fourth order in space and second-order in time staggered grid (SG) method for the solution of Biot's equation are presented. The numerical dispersion and stability conditions are derived. The scheme delivers stable results for heterogeneous media. Love-type wave propagation in different porous media is presented graphically. From different graphs, we can observe a difference in phase velocity for a different model. By observing the behaviour of the curve, we can understand the nature of the composite structure, which is essential for oil exploration. Also, our outcomes endorse that finite-difference modelling is a valuable tool to simulate wave propagation in poro-elastic media. So, in seismology, the deficiency of detailed velocity would be responsible for the error associated with computing wave propagation.

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