Analysis of Electrically Shorted and Open Cases on Love Waves Propagating in Piezoelectric Layer Embedded over a Fiber-reinforced Substrate

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Abstract

The article incisively analyzes the impact of piezoelectricity on Love wave transmission in an inhomogeneous bi-layered structure consisting of smoothly embedded thin piezoelectric material bonded to a semi-infinite fiber-reinforced medium. By applying the variable-separable method, a general form of dispersion equations, analyzing the Love waves' characteristics in electrically open and shorted cases of the piezoelectric material has been derived. The crux of the study lies in the fact that the presence of the prestresses in the upper layer and lower half-space along with elastic, piezoelectric, and permittivity coefficients lead the derived frequency relation to merge with the classical form of equations of the Love waves. The procured dispersion relation substantiates that the depth of the upper layer prestresses, and piezoelectricity coefficients play a guiding role in the transmission of Love waves. The numerical discussions and findings carry wider applications and may imply guidance of additive manufacturing of varied composite multi-materials for pre-stressed and microstructural configurations with partial and global dispersion properties.

Keywords: Love waves; Piezoelectricity; Fiber-reinforced; Electrically short case; Electrically open case

Introduction

The advent of piezoelectric materials which tend to hold unique characteristics to generate electric charge potentials subject to any form of mechanical stresses comprising compression, bending, and stretching leads to several engineering applications. Piezoelectric materials carry several utilitarian aspects in devices like sensors, actuators, filters, etc. To gain more insights into the localization properties of Love waves at piezoelectric surfaces, several researchers have worked on enhancing the sensitivity of SAW devices. Sensors remain highly sensitive in lieu of acoustic energy accumulation within specified wavelengths. In present times, the problems associated with the reflection and refraction phenomena of elastic waves have attracted paramount interest in the field of waves and acoustics [1-6]. Moreover, the plane wave transmission in anisotropic media traces some slightly different characteristics from those of inisotropic media. Li and Jin [7]) expressed how the SH- waves propagate in piezoelectric materials not smoothly bonded to a metal or elastic substrate. Chen et al. [8] illustrated the electromechanical characteristics of a piezoelectric actuator projected to a graded substrate including an adhesive material is studied with graphical representations. Chen et al. [9] investigated the interfacial imperfections of thin piezoelectric film clamped to a graded substrate. Chen et al. [10] developed the relations for interfacial behavior of a piezoelectric actuator lying over to a homogeneous half-space. Cooper and Reiss [11] described the incidence of reflection of harmonic plane waves on the free surface of a linearly viscoelastic semi-infinite space. The incidence of reflection and refraction of elastic waves on inhomogeneous earth media was extensively described by Nayfeh [12]. Du et al. [13] introduced how Love waves behave while propagating through functionally graded piezoelectric materials. Du et al. [14], in an analytical investigation, have shown the influence of prestress in piezoelectric layered structures underlying viscous liquid. Love waves involving a thin piezoelectric layer clamped smoothly to unbounded elastic subs. The propagation characteristics of Love-type waves in semi-infinite FGPM with quadratic variation were addressed by Eskandari and Shodja [15]. Liu and He [16] carried out extensive work to derive a general frequency equation for Love waves transmitting in a layered structure over a piezoelectric half-space. In addition to these studies, some unparalleled works have been nurtured by many authors discussing the problems relevant to piezoelectric materials and fiber-reinforced materials. The expressions for reflection coefficients of the reflected waves in lieu of P and SV waves on an insulated and isothermal-free surface were demonstrated by Othman and Song [17]. Tahir et al.[18,19] established the frequency relations for wave propagation for ceramic-metal functionally graded sandwich plates with different porosity distributions and functionally graded sandwich plates via a simple quasi-3D HSDT. Al-Furjan et al. [20] studied the fundamental properties of waves in a sandwiched structure with a soft core and multi-hybrid nano-composite (MHC) face sheets under higher-order shear deformable theory. Due to varied applications in smart devices, the reflection and transmission phenomena on piezoelectric and other complex materials are represented by several scientists and engineers across the globe. The incidence of reflection and transmission of plane waves at the interface of the piezoelectric and piezomagnetic substrates was analyzed by Pang et al. [21]. Moreover, Yuan and Zhu [22] studied the reflection and refraction of plane waves at an interface of two anisotropic piezoelectric substrates.

In the foundational work on piezoelectric plate vibrations, Tiersten [23] comprehensively explored linear vibrations in piezoelectric plates. This seminal publication by Tiersten in 'Linear Piezoelectric Plate Vibrations' from Plenum Press in New York, NY, significantly contributes to the understanding of piezoelectric plate dynamics.

In the exploration of guided waves, Nie, An, and Liu [24] extensively studied the characteristics of SH-guided waves in layered piezoelectric/piezomagnetic plates. In the context of characterizing fiber composites, Markham [25] made significant contributions to the field through the measurement of elastic constants using ultrasonics. This pivotal work by Markham on 'Measurement of the Elastic Constants of Fiber Composites by Ultrasonics' provides a crucial foundation for understanding the mechanical properties of such materials.

A comprehensive analysis of Free vibration, wave propagation, and tension analyses of a sandwich micro/nanorod subjected to electric potential using strain gradient theory was carried out by Arefi, M. and Zenkour, A.M., [26]. Arefi and Zenkour [27] conducted an insightful analysis of the wave propagation behavior of a functionally graded magneto-electro-elastic nanobeam resting on a Visco-Pasternak foundation. Their study contributes significantly to understanding the intricate dynamics of such nanostructures. In the study of microscale effects on structural behavior, Arefi and Zenkour [28] extensively investigated the influence of micro-length-scale parameters and inhomogeneities on the bending, free vibration, and wave propagation analyses of a functionally graded Timoshenko's sandwich piezoelectric microbeam. Their research significantly contributes to understanding the nuanced mechanical responses of such microstructures. Further Arefi, M. [29] worked on the Surface effect and non-local elasticity in wave propagation of functionally graded piezoelectric nano-rod excited to an applied voltage which has extensive implications on Love waves propagating in piezoelectric layer embedded over a fiber-reinforced substrate. In recent years, there has been a growing interest in the vibrational behavior of nanostructures, particularly those with piezoelectric properties. Arefi [30] conducted a comprehensive nonlocal free vibration analysis of a doubly curved piezoelectric nano-shell, shedding light on the intricacies of the dynamic response in such structures. This work serves as a foundation for understanding the unique characteristics of piezoelectric nanoshells, providing valuable insights that motivate our exploration into Love waves propagating in piezoelectric layer embedded over a fiber-reinforced substrate. As the field of structural analysis continually evolves, recent studies have explored advanced theories for a more accurate understanding of the mechanical behavior of materials. Arefi et al. [31] made notable contributions by applying the sinusoidal shear deformation theory and introducing the concept of a physical neutral surface in the analysis of functionally graded piezoelectric plates. Their work provides a sophisticated framework for investigating the intricate mechanical responses of such composite structures. In the realm of structural dynamics, the free vibration analysis of composite structures has garnered significant attention, particularly in the exploration of advanced materials. Arefi, Mohammad-Rezaei Bidgoli, and Zenkour [32] made a significant contribution by conducting a meticulous study on the free vibration analysis of a sandwich nano-plate. Their work, which includes a consideration of a functionally graded (FG) core and piezoelectric face sheets while accounting for the neutral surface, offers valuable insights into the dynamic behavior of such complex structures. In the exploration of wave propagation in nanomaterials, Arefi and Zenkour [33] made a significant contribution by employing coupled stress components and surface elasticity in the nonlocal solution of a functionally graded piezoelectric Love nano-rod model. This insightful work addresses the complexities associated with wave propagation in nanorods, laying the groundwork for a deeper understanding of the mechanical behavior in such intricate systems. In the realm of elasticity, Behdad and Arefi [34] proposed a mixed two-phase stress/strain-driven elasticity, presenting applications in static bending, vibration analysis, and wave propagation In the dynamic analysis of nanoscale structures, Behdad and Arefi [35] also had introduced a novel mixture model of elasticity, offering insights into buckling analysis, vibration behavior, and wave propagation in such structures. This pioneering work addresses fundamental aspects of nanoscale dynamics, providing a valuable framework for understanding the mechanical responses of these systems. In the area of plate reinforcement, Belfield, Rogers, and Spencer [36] explored stress distributions in elastic plates reinforced by fibers arranged in concentric circles. In the exploration of wave interactions at material interfaces, Das et al. [37] investigated the reflection and refraction of plane waves specifically at the loosely bonded common interface of piezoelectric fiber-reinforced and fiber-reinforced composite media. Their study provides valuable insights into the complex phenomena occurring at these interfaces.

In the investigation of size-dependent phenomena in micro/nanostructures, Dehsaraji, Arefi, and Loghman [38] conducted a thorough free vibration analysis of functionally graded piezoelectric micro/nanoshells. Their study, based on the modified couple stress theory and considering thickness stretching effects, contributes significantly to understanding the intricacies of size-dependent behaviors in these structures. Ezzin et al. [39] presented a comprehensive study on the propagation of guided waves in functionally graded magneto-electro elastic composites. Their work contributes significantly to the understanding of the complex behavior of guided waves in such composite materials.

In the investigation of SH-wave propagation in porous piezoelectric composites, Rakshit et al. [40] explored the effect of interfacial imperfections. Their study contributes valuable insights into the complex interplay between interfacial imperfections and wave propagation characteristics in these composite materials. In the context of wave propagation studies, Singh, Mistri, and Das [41] investigated the propagation of Love-type waves in a corrugated fiber-reinforced layer. Their research provides valuable insights into the dynamic behavior of such composite structures and in [42] Singh et al. employed a Green's function approach to study the influence of a point source. Their use of this approach provides a comprehensive understanding of the dynamic behavior of piezoelectric materials subjected to SH waves. A few path-breaking works on thermo-mechanical vibrations on piezoelectricity were carried out by Khorshidi and Karimi [43] and Khorshidi and Pagoli [44].

Beneath the earth, there are several pieces of evidence found that fiber-reinforced media may also co-exist with piezoelectric materials with elevated value of initial stress. The characteristic features of this type of material are different from the integral materials, namely concrete and steel. Due to its reduced weight and high strength, fiber-reinforced composites are used in various structural elevations. Inside composite materials, individual materials remain distinct. This material consists of the fiber, the matrix, and the interface between materials. PZT (piezoelectric ceramics) and PVDF (piezoelectric polymers), remained frequently usable in actuators and sensors for monitoring and repairing health materials. The authors have applied the separation of the variable method to analyze the dispersion relation in order to characterize the properties of the Love type waves in the aforementioned model and are validated with numerical results.

Formulation of problem

In this model, we have considered two media involving a piezoelectric layer and fiber-reinforced half-space under initial stresses P_1, P_2 respectively. Fiber-reinforced half-space is rested beneath a piezoelectric layer consisting of thickness h (in meters) (where $-h \le z \le 0$). The geometry of the model is tailored in such a way that the Love wave is transmitted along the x-axis and the z-axis is pointing vertically downwards as shown in Figure 1. The polarization direction of piezoelectric material is assumed to be along the direction of the y-axis.

The governing equations of piezoelectricity with prestress in the absence of body forces are given (Tiersten, [23])

$$\sigma_{ij,i} - \frac{1}{2}\left(P_1 u_{j,k}\right)_{,i} = \rho_0 \ddot{u}_j,\ D_{i,i} = 0,\ i, j = 1,2,3. \qquad (1)$$

in which σ_{ij} indicates stress tensor and D_i is the electric displacement, ρ_0 is the density of the material, and P_1 indicates the prestress. The subscript ‘,’ and a superimposed ‘.’ indicate the usual meaning of a partial derivative in respect of the coordinates and derivative with respect to time, respectively.

The governing relations for an anisotropic and linearly electro-elastic solid may be represented as

$$\sigma_{ij} = c_{ijkl}\gamma_{kl} - e_{kij}E_k,\ D_i = e_{ikl}\gamma_{kl} + \varepsilon_{ik}E_k \tag{2}$$

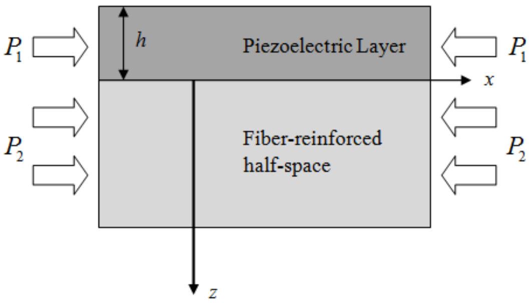

Fig. 1. Geometry of the problem

where γ_{kl} and E_k are the strain tensor and intensity of the electric field respectively. c_{ijkl}, e_{ikl} and ε_{ik} indicate the elastic, piezoelectric, and permittivity coefficients, respectively.

The strain-displacement relations are given by:

$$\gamma_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right),\ E_k = -\phi_{,k} \tag{3}$$

in which ϕ is the electric potential component and assuming the pre-stress P acting along the direction of the x-axis.

Displacement components and the electrical potential function are assumed to be

$$\begin{gathered} u(x,z,t) = 0,\ w(x,z,t) = 0, \\ v = v(x,z,t),\ \phi = \phi(x,z,t) \end{gathered} \tag{4}$$

in the case of Love wave transmitting horizontally and in which *u*, *v*, and *w* are the mechanical displacement components along the directions of the *x*, *y*, and *z*-axis respectively.

Solution for the Prestressed Piezoelectric substrate

On Substitution of equation (3) in equation (2) and in equation (1) with consideration on equation (4), the governing equations with respect to mechanical deformation and electrical potential turned into:

$$\left.\begin{aligned} c_{44}\nabla^2 v + e_{15}\nabla^2\phi - \frac{P_1}{2}\frac{\partial^2 v}{\partial x^2} = \rho_0 \frac{\partial^2 v}{\partial t^2} \\ e_{15}\nabla^2 v = \varepsilon_{11}\nabla^2\phi \end{aligned}\right\} \tag{5}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial z^2$ refers to the two-dimensional Laplace operator and ρ_0 refers to the density of the piezoelectric material

$$\begin{aligned} \sigma_{yz} &= c_{44}\frac{\partial v}{\partial z} + e_{15}\frac{\partial \phi}{\partial z}, \\ \sigma_{xy} &= c_{44}\frac{\partial v}{\partial x} + e_{15}\frac{\partial \phi}{\partial x} \end{aligned} \tag{6}$$

$$\begin{aligned} D_z &= e_{15}\frac{\partial v}{\partial z} - \varepsilon_{11}\frac{\partial \phi}{\partial z}, \\ D_x &= e_{15}\frac{\partial v}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x} \end{aligned} \tag{7}$$

The substitution

$$\psi = \phi - \frac{e_{15}}{\varepsilon_{11}}v \tag{8}$$

reduces equation (5) into

$$\left.\begin{aligned} c_{44}^{*}\nabla^2 v - \frac{P_1}{2}\frac{\partial^2 v}{\partial x^2} = \rho_0 \frac{\partial^2 v}{\partial t^2} \\ e_{15}\nabla^2\psi = 0 \end{aligned}\right\} \tag{9}$$

where

$$c_{44}^{*} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}.$$

we consider the following solution of equation (9)

$$\begin{aligned} v &= v_1(z)\exp\left[ik(x-ct)\right], \\ \psi &= \Psi(z)\exp\left[ik(x-ct)\right] \end{aligned} \tag{10}$$

where *k* and *c* are the wavenumber and phase velocity respectively, $v_1(z)$ and $\Psi(z)$ are the respective solutions of

$$\left.\begin{aligned} \frac{d^2 v_1}{dz^2} + k^2 b_1^2 v_1(z) = 0 \\ \frac{d^2 \Psi}{dz^2} - k^2 \Psi(z) = 0 \end{aligned}\right\} \tag{11}$$

where

$$b_1^2 = \left(\frac{c^2}{c_0^2} + \frac{P_1}{2c_{44}^*} - 1\right) \text{and } c_0 = \sqrt{\frac{c_{44}^*}{\rho_0}} \;.$$

So the solutions of equation (11) are

$$\left.\begin{aligned} v_1(z) = C_1 \cos(kb_1 z) + C_2 \sin(kb_1 z) \\ \Psi(z) = C_3 e^{kz} + C_4 e^{-kz} \end{aligned}\right\} \tag{12}$$

where C_1, C_2, C_3, C_4 are arbitrary constants.

Therefore the displacement components and the electric potential can be written as:

$$v_1(x,z,t) = \left[C_1 \cos(kb_1 z) + C_2 \sin(kb_1 z)\right] \exp\left[ik(x - ct)\right] \tag{13}$$

$$\phi(x,z,t) = \left[\frac{e_{15}}{\varepsilon_{11}}\{C_1 \cos(kb_1 z) + C_2 \sin(kb_1 z)\} + C_3 e^{kz} + C_4 e^{-kz}\right] \exp\left[ik(x - ct)\right] \tag{14}$$

Solution of Pre-stressed fiber-reinforced half-space

The constitutive equation for a fiber-reinforced linearly elastic anisotropic medium is given by(Belfield et al. [36])

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_k a_i e_{kj} + a_k a_j e_{ki}) + \beta(a_k a_m e_{km} a_i a_j)$$

$$, i, j, k, m = 1,2,3 *$$

Where τ_{ij} are stress components, $e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ are the strain components, δ_{ij} indicates the Kronecker delta.

$\vec{a} = (a_1, a_2, a_3)$ refers to the preferred direction of reinforcement so that $a_1{}^2 + a_2{}^2 + a_3{}^2 = 1$. The vector $\vec{a}$ may be a function of position where the indices assume values from 1,2,3. Here α , β and $(\mu_L - \mu_T)$ represents reinforced parameters. μ_T identified as the shear modulus in transverse shear along the preferred direction, μ_L refers to the shear modulus in longitudinal shear in the preferred direction. α , β remains as specific stress components relevant to dissimilar layers for the concrete part of the composite material and λ is Lame's constant.

Thus the equation of motion for the propagation of SH waves is represented as

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} - \frac{P_2}{2}\frac{\partial \omega_{21}}{\partial x} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{15}$$

where

$$\tau_{12} = \tau_{21} = \left[\mu_T \frac{\partial v_2}{\partial x} + (\mu_L - \mu_T)\left(a_1^2 \frac{\partial v_2}{\partial x} + a_1 a_3 \frac{\partial v_2}{\partial z}\right)\right]$$

$$\tau_{23} = \tau_{32} = \left[\mu_T \frac{\partial v_2}{\partial z} + (\mu_L - \mu_T)\left(a_1 a_3 \frac{\partial v_2}{\partial x} + a_3^2 \frac{\partial v_2}{\partial z}\right)\right]$$

and $\omega_{21} = \frac{1}{2}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right) = \frac{1}{2}\frac{\partial v_2}{\partial x}$ are the rotational components in half space.

Therefore equation (15) becomes

$$\begin{aligned} &\mu_T \frac{\partial^2 v_2}{\partial x^2} + (\mu_L - \mu_T)(a_1^2 \frac{\partial^2 v_2}{\partial x^2} + a_1 a_3 \frac{\partial^2 v_2}{\partial x \partial z}) + \\ &\mu_T \frac{\partial^2 v_2}{\partial z^2} + (\mu_L - \mu_T)(a_1 a_3 \frac{\partial^2 v_2}{\partial z \partial x} + a_3^2 \frac{\partial^2 v_2}{\partial z^2}) - \\ &\frac{P_2}{4}\frac{\partial^2 v_2}{\partial x^2} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \end{aligned} \tag{16}$$

which becomes

$$P\frac{\partial^2 v_2}{\partial z^2} + (Q + \xi_1)\frac{\partial^2 v_2}{\partial x^2} + R\frac{\partial^2 v_2}{\partial x \partial z} = \frac{1}{c_2^2}\frac{\partial^2 v_2}{\partial t^2} \tag{17}$$

where

$$P = 1 + \left(\frac{\mu_L}{\mu_T} - 1\right)a_3^2, \quad Q = 1 + \left(\frac{\mu_L}{\mu_T} - 1\right)a_1^2,$$

$$R = 2a_1 a_3\left(\frac{\mu_L}{\mu_T} - 1\right), \ \xi_1 = -\frac{P_2}{4\mu_T}, \ c_2^2 = \frac{\mu_T}{\rho_2}$$

Assuming the solution of equation (17) in the form of

$$v_2(x,z,t) = v_2(z)e^{ik(x-ct)} \tag{18}$$

where k and c indicate the wavenumber and wave velocity respectively.

Solving equation (17) becomes

$$\frac{d^2 v_2}{dz^2} + \left(\frac{ikR}{P}\right)\frac{dv_2}{dz} - \frac{1}{P}\left\{(Q+\xi_1)k^2 - \frac{c^2k^2}{c_2^2}\right\}v_2(z) = 0 \tag{19}$$

The solution of equation (19) yields the expression for the non-vanishing displacement component of half-space as $v_2(z) = Ce^{-P^* z}e^{ik(x-ct)}$

where $P^* = \frac{ikR}{2P} + k\sqrt{\left(\frac{Q+\xi_1}{P}\right) - \frac{R^2}{4P^2} - \frac{c^2}{c_2^2 P}}$

(C is an arbitrary constant)

Boundary conditions:

Mechanical boundary conditions:

(a) Referring to the free surface as devoid of stress, *i.e.*, $\sigma_{yz}(x,-h,t) = 0$

(20)

(b) Stress and mechanical displacement remained continuous, *i.e.*, $\sigma_{yz}(x,0,t) = \sigma_{yz}^s(x,0,t)$

and $v_1(x,0,t) = v_2(x,0,t)$

(21)

Electrical boundary conditions:

(a) As the free surface is free from any electric potential, *i.e.*, $\phi(x,0,t) = 0$

(22)

(b) At z= -*h*, electrically open and electrically shorted, are considered:

For electrically open case: electrical displacement turns into zero, *i.e.*, $D_z(x,-h,t) = 0$ (23)

and for electrically shorted case: electric potential turns to be zero, *i.e.*, $\phi(x,-h,t) = 0$ (24)

Using the equations (13) and (14) and all the above boundary conditions, we get

$$C_1 b_1 c_{44}^* \sin(kb_1h) + C_2 b_1 c_{44}^* \cos(kb_1h) + C_3 e_{15} e^{-kh} - C_4 e_{15} e^{kh} = 0$$

$$C_2 c_{44}^* b_1 k + C_3 k e_{15} - C_4 k e_{15} - C\left\{(\mu_L - \mu_T)\left(a_1 a_3 ik - a_3^2 p^*\right) - \mu_T p^*\right\} = 0$$

$$C_1 = C$$

$$\frac{e_{15}}{\varepsilon_{11}} C_1 + C_3 + C_4 = 0$$

$$C_3 e^{-kh} - C_4 e^{kh} = 0$$

$$\frac{e_{15}}{\varepsilon_{11}} C_1 \cos(kb_1h) - C_2 \frac{e_{15}}{\varepsilon_{11}} \sin(kb_1h) + C_3 e^{-kh} + C_4 e^{kh} = 0$$

where $c_{44}^* = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$

Dispersion relation for electrically open case

Solving (25) to (29) we get

$$c_{44}^{*}b_1 k\tan(kb_1h)+\frac{e_{15}^2}{\varepsilon_{11}}k\tanh(kh)+$$
$$(\mu_L-\mu_T)(a_1a_3ik)-\left\{a_3^2(\mu_L-\mu_T)+\mu_T\right\}.\frac{ikR}{2P}$$
$$+\left\{a_3^2(\mu_L-\mu_T)+\mu_T\right\}\left(k\sqrt{\left(\frac{Q+\xi_1}{P}\right)-\frac{R^2}{4P^2}-\frac{c^2}{C_2^2P}}\right)=0 \quad (31)$$

whose real part is

$$c_{44}^{*}b_1 k\tan(kb_1h)+\frac{e_{15}^2}{\varepsilon_{11}}k\tanh(kh)+$$
$$\left\{a_3^2(\mu_L-\mu_T)+\mu_T\right\}\left(k\sqrt{\left(\frac{Q+\xi_1}{P}\right)-\frac{R^2}{4P^2}-\frac{c^2}{C_2^2P}}\right)$$

And imaginary part is

$$(\mu_L-\mu_T)(a_1a_3k)-\left\{a_3^2(\mu_L-\mu_T)+\mu_T\right\}.\frac{kR}{2P}$$

Dispersion relation for electrically sorted case

Solving (25)-(28) and (30) we get

$$A+B+C+D=0$$
$$A=-\left\{(\mu_L-\mu_T)(a_1a_3ik-a_3^2P_2)+\mu_TP_2\right\}\left\{(e^{kh}-e^{-kh})b_1c_{44}^{*}\cos(kb_1h)-\frac{e_{15}^2}{\varepsilon_{11}}\sin(kb_1h)(e^{kh}+e^{-kh})\right\}$$
$$B=2b_1c_{44}^{*}\sin^2(kb_1h)\frac{e_{15}^2}{\varepsilon_{11}}k+b_1^2c_{44}^{*\,2}k\sin(kb_1h)(e^{-kh}-e^{kh})$$
$$C=-\frac{e_{15}^2}{\varepsilon_{11}}kb_1c_{44}^{*}\cos(kb_1h)(e^{kh}+e^{-kh})+\frac{2e_{15}^2}{\varepsilon_{11}}kb_1c_{44}^{*}+\frac{e_{15}^4}{\varepsilon_{11}^2}k\sin(kb_1h)(e^{kh}-e^{-kh})$$
$$D=\frac{2e_{15}^2}{\varepsilon_{11}}k\cos^2(kb_1h)b_1c_{44}^{*}-\frac{e_{15}^2}{\varepsilon_{11}}\cos(kb_1h)c_{44}^{*}kb_1(e^{kh}+e^{-kh})$$

Particular cases

Case I

If we remove initial stresses from the piezoelectric layer and fiber-reinforced half space i.e $P_1\to 0$ and $P_2\to 0$ then the equation becomes

$$b_{11}c_{44}^{*}k\tan(kb_{11}h)-A_{25}^{'}+\frac{e_{15}^2}{\varepsilon_{11}}k\tanh kh=0 \quad (33)$$

where $b_{11}^2=\frac{c^2}{c_0^2}-1$

$$A_{25}^{'}=-\left[(\mu_L-\mu_T)(a_1a_3ik-a_3^2P_1^{*})-\mu_TP_1^{*}\right]$$
$$P_1^{*}=\frac{ikR}{2P}+k\sqrt{\frac{Q}{P}-\frac{R^2}{4P^2}-\frac{c^2}{C_2^2P}}$$

and

$$A_1+B_1+C_1+D_1=0$$
$$A_1=-\left\{(\mu_L-\mu_T)(a_1a_3ik)\right\}.$$
$$\left\{(e^{kh}-e^{-kh})b_{11}c_{44}^{*}\cos(kb_{11}h)-\frac{e_{15}^2}{\varepsilon_{11}}\sin(kb_{11}h)(e^{kh}+e^{-kh})\right\}$$
$$B_1=2b_{11}c_{44}^{*}\sin^2(kb_{11}h)\frac{e_{15}^2}{\varepsilon_{11}}k+$$
$$b_{11}^2c_{44}^{*\,2}k\sin(kb_{11}h)(e^{-kh}-e^{kh})$$
$$C_1=-\frac{e_{15}^2}{\varepsilon_{11}}kb_{11}c_{44}^{*}\cos(kb_{11}h)(e^{kh}+e^{-kh})+$$
$$(32)\ \frac{2e_{15}^2}{\varepsilon_{11}}kb_{11}c_{44}^{*}+\frac{e_{15}^4}{\varepsilon_{11}^2}k\sin(kb_{11}h)(e^{kh}-e^{-kh})$$
$$D_1=\frac{2e_{15}^2}{\varepsilon_{11}}k\cos^2(kb_{11}h)b_{11}c_{44}^{*}-$$
$$\frac{e_{15}^2}{\varepsilon_{11}}\cos(kb_{11}h)c_{44}^{*}kb_{11}(e^{kh}+e^{-kh})$$

Case II

For a demonstration of the frequency equation of Love waves in a prestressed piezoelectric layer over a prestressed isotropic & homogeneous half-space, we make $\mu_L=\mu_T\to\mu_2$. Then both equations become

$$b_{12}c_{44}^{*}k\tan(kb_{12}h)-A_{25}^{''}+\frac{e_{15}^2}{\varepsilon_{11}}k\tanh kh=0 \quad (35)$$

Where $b_{11}^2 = \frac{c^2}{c_0^2} + \frac{P_1}{2c_{44}^*} - 1$

$$A_{25}^* = \mu_2 P_2^*$$

$$P_2^* = k\sqrt{(1+\xi_{11}) - \frac{c^2}{C_{21}^{2*}P}}$$ and

$$A_2 + B_2 + C + D = 0$$

$$A_2 = \mu_2 P_2 [b_{12} c_{44}^* \cos(kb_{12}h)(e^{kh} - e^{-kh}) - \frac{e_{15}^2}{\varepsilon_{11}} \sin(kb_{12}h)(e^{-kh} + e^{kh})$$

$$B_2 = 2b_{12} c_{44}^* \sin^2(kb_{12}h) \frac{e_{15}^2}{\varepsilon_{11}} k + b_{12}^2 c_{44}^{*\,2} k \sin(kb_{12}h)(e^{-kh} - e^{kh})$$

$$C_2 = -\frac{e_{15}^2}{\varepsilon_{11}} kb_{12} C_{44}^* \cos(kb_{12}h)(e^{kh} + e^{-kh}) + \frac{2e_{15}^2}{\varepsilon_{11}} kb_{12} C_{44}^* + \frac{e_{15}^4}{\varepsilon_{11}^2} k \sin(kb_{12}h)(e^{kh} - e^{-kh})$$

$$D_2 = \frac{2e_{15}^2}{\varepsilon_{11}} k \cos^2(kb_{12}h) b_{11} C_{44}^* - \frac{e_{15}^2}{\varepsilon_{11}} \cos(kb_{12}h) C_{44}^* kb_{12} (e^{kh} + e^{-kh})$$

Case III

For derivation of the frequency relation of the Love waves in an isotropic, homogeneous substrate embedded on an inhomogeneous semi-infinite space in the absence of prestress, and reducing the piezoelectric coefficient into zero, i.e., $e_{15} \to 0$, $c_{44}^* = c_{44} \to \mu_1$, then the dispersion equation of (31) and (32) become in both cases as,

$$b_{13}\mu_1 k \tan(kb_{13}h) - A_{25} = 0 \quad (37)$$

where $b_{13}^2 = \frac{c^2}{c_0^{'2}} + \frac{P_1}{2\mu_1} - 1$

and

$$A_3 + B_3 = 0$$

$$A_3 = -\{(\mu_L - \mu_T)(a_1 a_3 ik - a_3^2 P_2)\}. \{(e^{kh} - e^{-kh}) b_{13}\mu_1 \cos(kb_{13}h)\}$$

$$B_3 = b_{13}^2 \mu_1^2 k \sin(kb_{13}h)(e^{-kh} - e^{kh})$$

$$C_3 = 0$$

$$D_3 = 0$$

(36)

Case IV

The dispersion equations of Love wave in prestressed isotropic, homogeneous layer over prestressed isotropic & homogeneous half-space is obtained by considering $e_{15} \to 0$, $c_{44}^* = c_{44} \to \mu_1$, $\mu_L = \mu_T \to \mu_2$.

$$b_{14}\mu_1 k \tan(kb_{14}h) - A_{25}''' = 0 \quad (39)$$

where

$$b_{14}^2 = \frac{c^2}{c_0^{''2}} + \frac{P_1}{2\mu_1} - 1,\ c_0 = c_0'' = \sqrt{\frac{\mu_1}{\rho_0}},\ c_2^2 = \frac{\mu_T}{\rho_2},\ A_{25}''' = \mu_2 P_3^*$$

and

$$A_4 + B_4 = 0$$

$$A_4 = \mu_2 P_2 [b_{14}\mu_1 \cos(kb_{14}h)(e^{kh} - e^{-kh})$$

$$B_4 = b_{14}^2 \mu_1^2 k \sin(kb_{14}h)(e^{-kh} - e^{kh})$$

Case V

If we neglect the prestress in *Case IV* i.e., by setting $e_{15} \to 0$, $c_{44}^* = c_{44} \to \mu_1$, $\mu_L = \mu_T \to \mu_2$ and $P_1 \to 0, P_2 \to 0$, we get the dispersion equation as follows:

$$b_{15}\mu_1 k \tan(kb_{15}h) - A_{25}^{iv} = 0 \quad (41)$$

where

$$b_{15}^2 = \frac{c^2}{c_0^{'''2}} - 1$$

$$c_0 = c_0^{''} = \sqrt{\frac{\mu_1}{\rho_0}}$$

$$c_2^2 = \frac{\mu_T}{\rho_2}$$

$$A_{25}^{iv} = \mu_2 P_4^*$$

and

$$B_5 = 0$$

$$B_5 = b_{15}^2 \mu_1^2 k \sin(kb_{15}h)(e^{-kh} - e^{kh}) \quad (42)$$

Therefore, for both cases, the dispersion equations (31) and (32) reduce to the following:

$$\tan\left(kh\sqrt{\frac{c^2}{c_0^2} - 1}\right) = \frac{\mu_2}{\mu_0} \frac{\sqrt{1 - \frac{c^2}{C_2^2}}}{\sqrt{\frac{c^2}{C_2^2} - 1}}$$

The above equation is the classical form of the Love Equation in a stress-free piezoelectric layer lying over a stress-free fiber-reinforced half-space.

Numerical calculations & Discussions:

For studying the impact of prestress on the propagation of Love waves in a piezoelectric substrate embedded over fiber-reinforced half-space, the values $\frac{c}{c_0}$ from equations (31) and (32) have been derived and graphically represented and illustrated. The figures represent the variation of non-dimensional phase velocity with respect to the dimensionless wave number using the relevant values from Table 1 and Table 2.

Table 1. Values of Material coefficients of the piezoelectric material(PZT-4) (Nie et al. [24])

$c_{44}(\times10^9 N/m^2)$	$e_{15}(C/m^2)$	$\varepsilon_{11}(\times10^{-9} C^2/Nm^2)$	$\rho(kg/m^3)$
25.6	12.7	6.45	7500

Table 2. The following data has been taken into consideration for the fiber-reinforced half space under initial stress P_2 (Markham [25]):

$\mu_L(\times10^{10} N/m^2)$	$\mu_T(\times10^{10} N/m^2)$	$\rho_2(kg/m^3)$
7.07	3.50	1600

For the numerical computation, we have considered the thickness (h) of the piezoelectric layer as 1.0, 1.25, 1.5, and 1.75 respectively. The relevant curves in the above figure 2 reflect that gradual elevation of the thickness of the piezoelectric layer tends to increase the phase velocity of the transmitting Love wave gradually which means that the thickness of the upper layer has a significant effect on Love wave phase velocity.

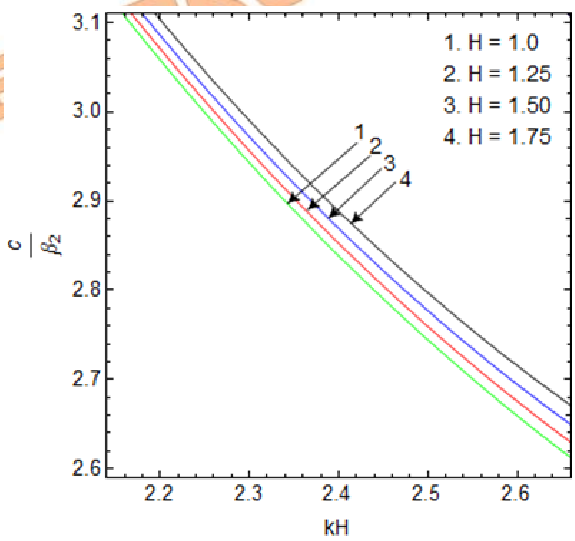

Fig. 2. Variation of Dimensionless phase velocity against dimensionless wavenumber for different values of depth

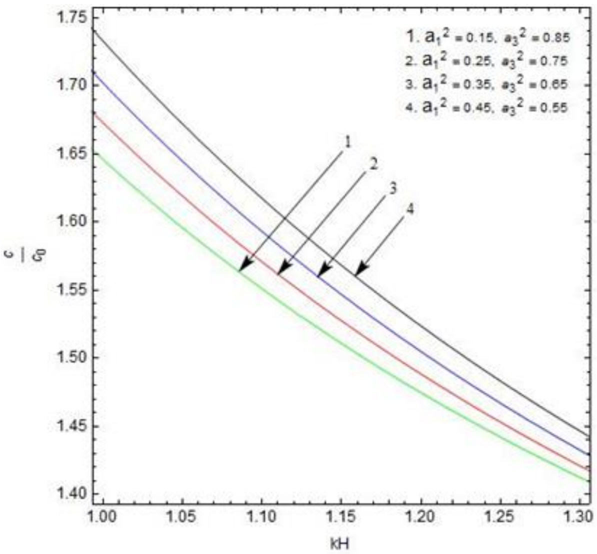

Fig. 3. Variation of Dimensionless phase velocity w.r.t dimensionless wavenumber with different values of a_1 and a_3 where $a_2 = 0$. From this figure, we can say that with the increased value of

a_1 and decrease the value of a_3, phase velocity also increases.

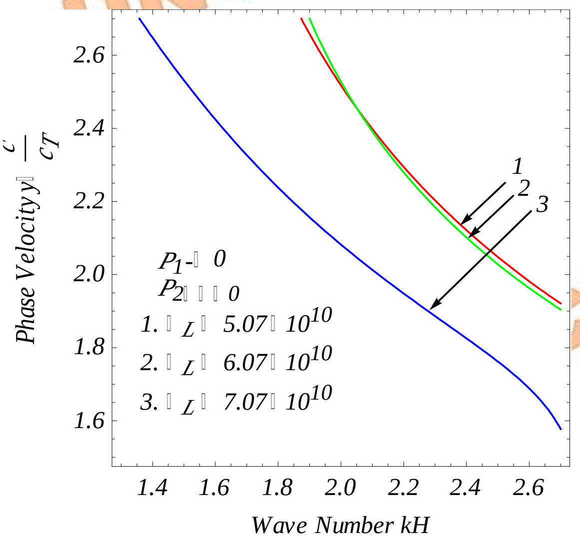

Fig. 4. Variation of Dimensionless phase velocity w.r.t dimensionless wavenumber with different values of μ_L and where both the prestresses P_1 and P_2→ 0.

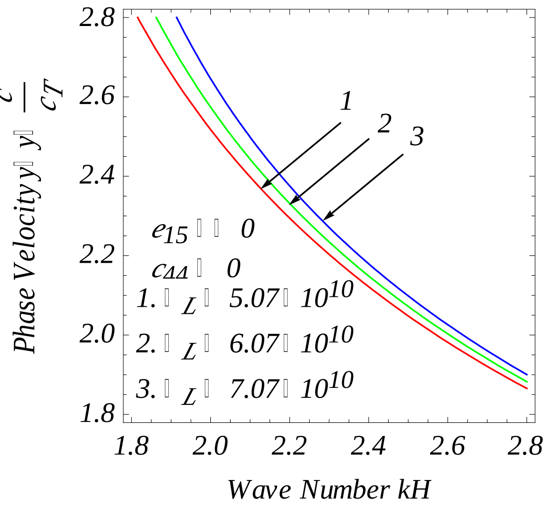

Fig. 5. Variation of Dimensionless phase velocity w.r.t dimensionless wavenumber with different values of μ_L and where both $e_{15} \to 0 \, and \, c_{44} \to \mu_1$.

From this figure, we can say that with the increase value of μ_L, phase velocity also increases.

The curves in the above figure 3 show that the increasing values of the reinforcement parameter a_1 and accordingly decreasing values of the reinforcement parameter a_3, resulted in an increase in the phase velocity of the Love waves. It is clear from this figure that an increase in the reinforcement parameter a_1 bounded with the expression $a_1^2 + a_3^2 = 1$ favors the phase velocity of Love waves by inflating it.

Conclusions:

Due to intrinsic lightweight properties and having high material strength and stiffness, fiber-reinforced composites can be used in utilitarian aspects like industries relevant to construction, aviation, and medical instruments. The materials can significantly reduce damages in adjoining surfaces even due to severe vibrational impacts. In spite of having a wide range of applications in various engineering applications, piezoelectric materials carry drawbacks like reduced levels of piezoelectric constants, shape control, and high acoustic impedance which may result in lowering the control on derived distributed actuators. Keeping view of that to improve the efficiency levels, the proposed model of fiber-reinforced piezoelectric composite materials with the presence of prestress has been studied for both electrically open and shorted cases and thereby the dispersion relations have been established with the aid of variable-separable method by which we can easily eliminate the first-order differential terms for further simplification. From the dispersion relation and subsequent graphs, the outcomes can be briefed as:

(i) The depth of the upper layer carries a significant effect on the phase velocity of the Love waves for the model

(ii) the increasing values of reinforcement parameters with the given restrictive condition show that phase velocity increases for any fixed wavenumber for increasing values of the reinforcement parameter a_1.

(iii) the increasing rigidity of the half-space and the piezoelectric constants often impedes the transmission of Love waves to a certain extent within a specified range of wave numbers.

In view of the consequences drawn from the study, the results are useful for designing high-mode Love mode sensors and can also be used for developing medical imaging instruments and underwater transducers. Precisely, the present study is likely to prove its efficiency in modeling the damages caused by the artificial explosion of the layered piezoelectric reinforced composite structures. Since the properties of SAW devices specifically depend on phase delay, therefore, from a utilitarian aspect the outcomes can be employed in the designing of Love wave sensors.

Data Availability Statement

No specific data information is available

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