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Crushing Analysis of Empty and Foam-Filled Cylindrical and Conical Corrugated Composite Tubes

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ABSTRACT

In the present paper, a numerical study is performed to investigate the crushing behavior of cylindrical and conical corrugated composite tubes. Different aluminum foams are applied to proposed structures in order to offer an excellent energy absorption capacity. The crushing behavior of tubes are evaluated by two parameters: SEA (specific energy absorption) and CFE (crush force efficiently). So, in order to study the effect of corrugation on the crashworthiness of composite tubes, a comprehensive numerical analysis of corrugated carbon/BMI tubes is performed under axial loadings in this work. The effect of geometric parameters of corrugation including number and radius of corrugations is studied by FE simulation of several models in LS-DYNA. Comparison between corrugated tubes and straight one is demonstrated the CFE is improved significantly in these new models. Furthermore the absorbed energy is increased by using foams. SEA, mean force and peak forces are increased by increasing the foam density while the crush force efficiently is decreased considerably due to the fact that in higher densities, densification region accrues in fewer strains.

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1. Introduction

In recent years, the interest of using energy absorber devices with higher crashworthiness capacity has been increased [1]. Among various available models, thin-walled structures are known as the imperative components for energy absorption and therefore they play an important role in industrial transportation systems. The crush boxes are fabricated by a wide range of materials such as aluminum, steel and composites which are used in different shapes such as circular, triangular, conical, squared and polygonal tubes [2-7]. The extensive researches concerning the crashworthiness of en-ergy absorbers demonstrate that composite tubes are one of the widespread structures used in vari-ous topologies due to their low density and high energy absorption capacity. Studies state that a composite structure can display higher energy absorption than that of metals [8, 9]. However, the energy absorption of composite structures depends on a wide range of factors such as material characteristics, ply design, geometry, etc and a considerable amount of literature has been published on the energy absorption characteristics of composite structures affected by these factors [10-14]. Due to the recent developments of composite manufacturing methodologies, carbon fiber reinforced polymer (CFRP) components are widely used and fabricated in automotive and aerospace industries [10-13].

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 Table 1. Number and radius of curvatures of corrugated models

 (b) to (e) in Figs. 1 and 2.

Code	r (mm)	n
Cr18n8	18	8
CCr18n8	18	8
Cr9n12	9	12
CCr9n12	9	12
Cr5n16	5	16
CCr5n16	5	16
Cr3n20	3	20
Cr3n20	3	20

2.2. Finite element modeling

The present numerical investigation is performed using a non-linear explicit dynamic LS-DYNA code. As illustrated in Fig. 3 and 4 the FE model consists of three main parts as: 1) the corrugated composite tube, 2) aluminum foam and 3) the mass block. The mass block is modeled as a rigid body by 'RIGID_MAT' in LS-DYNA. It should be noted that the mass is considered 500 kg and also young's modulus is assigned as 200GPa. It is also worth mentioning that the mass block is allowed to move in the z-axis only. The material model **MAT54**, 'ENHANCED_COMPOSITE_DAMAGE,' progressive failure damage, is used to simulate the mechanical

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composite materials crashworthiness, simple axisymmetric geometries were focused including revolutionary surfaces such as circular, elliptical, and conical surfaces, and flat surfaces such as square, rectangular, etc. Recently the corrugations are introduced in the tube to force the plastic deformation to occur at predetermined intervals along the tube generator as an effective and innovative solution [15-18]. The aims are to improve the uniformity of the load-displacement behavior of axially crushed tubes, predict and control the mode of collapse in each corrugation in order to optimize the energy absorption capacity of the tube. Eyvazian et al. [19] experimentally investigated the effect of corrugation on crushing behavior and energy absorption of aluminum circular tubes. The results displayed that tubes with corrugation have a uniform force-displacement curve without an initial peak load. Alkhatib et al. [20] numerically studied the crushing behavior and performance of corrugated tapered tubes (CTT) under oblique loading conditions. It was found that some CTTs can achieve higher specific energy absorption relative to their tapered conventional counterparts and increasing the impact angles lead to a reduction in performance. Elgalai et al. [21] experimentally analyzed crushing of composite corrugated tubes subjected to quasi-static loading. Results confirmed that changing corrugation angle and fiber type enhance the energy absorption performance of composite tube. Numerical study of energy absorption of cotton fibre/propylene corrugated tubes [22] also displays that the tube's energy absorption capability was affected significantly by varying the number of corrugation and aspect ratios. It is found that as the number of corrugations increases, the amount of absorbed energy significantly increases. Crushing behavior of corrugated metal-composite tube was ex-

In a considerable number of papers related to

In this work, a comprehensive numerical analysis of cylindrical and conical corrugated car-bon/Bismaleimide (BMI) tubes are performed under axial loadings. To do this, different tubes by varying the radius and number of curvatures are modeled and analyzed using LS-DYNA explicit dynamic code. These models are validated with appropriate experimental and analytical solutions. Performing a parametric study on geometrical corrugation parameters of tubes indicates that the energy absorption of the structures depends strongly on the corrugation parameters. Based on the obtained results, generating corrugated surfaces on tubes improves the crush force efficiency significantly.

amined experimentally under axial loading condi-

tion by Eyvazian et al. [23].

2. Numerical modeling

2.1. Description of the models

This study proposes corrugated cylindrical and conical composite tubes in order to offer better energy absorption in comparison with conventional straight tubes. It is interesting to note that corrugation is formed in rows of wavelike folds or basically shaped into a series of regular folds that look like waves [22]. In all cases, tubes have 110 mm in length and 2 mm in thickness. In cylindrical tubes, the diameter is 60 mm and the upper and lower diameters are 44 mm and 76 mm respectively in conical tubes. Corrugation is defined by the number of wave curvatures, n and the radius of curvatures, r. Each concave or convex of tube determines one curvature.

In order to analyze the effect of corrugation on crushing behavior of composite tubes, various types of corrugated composite carbon/BMI tubes have been compared with a non-corrugated one as a standard structure. The mentioned tubes are shown in Figs. 1 and 2 for cylindrical and conical models respectively. The number and also the radius of curvatures in tubes are introduced by a code as expressed in Table 1.

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convergence analysis for a foam-filled composite tube leads to considering the element size of 2 *mm* and 5 *mm* for shells and foam, respectively.

2.3. Material Properties of the models

In proposed structures, all the specimens consist of 16 plies of T700/QY8911 (carbon fiber T700 reinforced BMI resin QY8911) which are modeled by "COMPOSITE_PART" code in LS-DYNA. In order to simulate the aluminum foam, the material model MAT_154 "DESHPANDLE_FLECK_FOAM" is used in this paper. MAT_154 is an appropriate material model for simulation of aluminum foam that is used as the filler material in energy absorbers [6]. The effect of foam density was investigated by comparing the results of structures with different foam densities of 220 and 534 and 710 g/cm³. The mechanical properties of composite and foam filler which are used in simulation are tabulated in Tables 2 and 3 respectively.

Table 2. Material properties of unidirectional carbon fiber/BMI
resin lamina [25].

$\rho(g/cm^3)$	$E_a(GPa)$	$E_b(GPa)$	$G_{ab}(GPa)$	ϑ_{ba}
1.53	135	9.12	5.67	0.021
$X_t(MPa)$	$X_c(MPa)$	$Y_t(MPa)$	$Y_c(MPa)$	$S_c(MPa)$
2326	1236	51	209	87.9

Table 3. The mechanical properties of aluminium foam [26]

	$ ho_f$	σ_p	α2	2	r	e e
	(g/cm ³)	(N/mm^2)	(N/mm^2)	P	(N/mm^2)	ε _D
	0.220	2.14	169	2.94	2.45	2.507
11	0.534	12.56	1544	3.68	1.00	1.620
L	0.710	22.18	4295	4.71	6.438	1.335

3. Crashworthiness parameters

First of all, it is necessary to introduce the primary parameters that are used to evaluate the crushing behavior of energy absorber structures. The most



important factor is the absorbed energy (EA) that is defined as below [26]:

$$EA = \int_0^{\delta_{max}} Fd\delta$$

(1)

Where F, δ and δ_{max} represent crushing force, axial displacement and the maximum deflection of structure, respectively.

SEA (Specific Energy Absorption), defined as the ratio of the absorbed energy to the mass of the structure, is another vital factor which is applied for comparison among different specimens [26].

$$SEA = EA/M \tag{2}$$

Furthermore, initial peak force, and mean force are the maximum force occurred during crushing and the ratio of absorbed energy to maximum deflection, respectively. Mean Force is calculated by the following equation [26]:

$$F_m = EA/\delta_{max} \tag{3}$$

By considering the mentioned parameters, Crushing Force Efficiency (CFE) is defined as the ratio of mean force to peak force [26]:

$$FFE = F_m / F_{max} \tag{4}$$

Where F_m and F_{max} are the mean force and peak force respectively. It should be demonstrated that the higher values of SEA and CFE increase energy absorption and safety of structures.

4. Numerical results and discussion

4.1. Validation

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4.1.1. Validation of composite tube

In this paper, a circular composite tube that has been evaluated in [27] is validated based on the experimental test. In this test, uniform length of 125 mm and an internal diameter of 50 mm were considered for G803/5224 specimen. A triggered mechanism is also set up at the top of the structure. Figs. 5 and 6 compare the force-displacement curve and final deformation pattern of the specimen between FE simulation and experiment, respectively. According to Fig. 6, FE simulation can predict final deformation pattern of the composite tubes. Also, Fig. 5 shows suitable agreement between experimental and numerical curves.



Figure 5. Force-displacement curves of the composite tube from numerical and experimental [27] results.



Figure 6. Numerical and experimental [27] final deformation pattern of the composite tube.

4.1.2. Validation of aluminium foam

In this section, the proposed FE model of an aluminum tube filled with aluminum foam is compared with experimental and theoretical results which are indicated in [28, 29]. In order to validate the numerical results, the experiment test which is done in [27], and the related theoretical solution is applied. On the other hand, according to the above approach, mean force and peak force of the model is calculated theoretically. Table 4 shows a comparison between numerical, experimental and theoretical mean force

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and peak force of the model. Obviously, there is suitable agreement among numerical results and both of experimental and theoretical solution.

theoretical mean force and peak force of the model.					
Force (kN)	FEM	Experiment	Analytical		
Mean force	41.26	41.5	38.6		
Peak force	58.63	53	51.9		

4.2. Crashworthiness of empty corrugated tubes

Fig. 7 depicts the force-displacement curves for empty straight and corrugated cylindrical tubes subjected to axial crushing. As indicated in Fig. 7, the straight cylindrical tube experienced a sharp drop in force at the beginning of the crushing that leads to less CFE in comparison with corrugated ones. As shown in Fig. 7, peak force of straight tube is higher than corrugated tubes significantly. It is clear from Fig. 7 that the maximum deflection is about 65 mm for straight tube while it is more than 80 mm for corrugated ones. Hence corrugation improves the crushing behavior of tubes. On the other hand, a comparison of SEA between corrugated tubes and straight one demonstrates that straight tube has the highest value of SEA. However, corrugation improves CFE without an intense reduction in the SEA. Also, the crashworthiness parameters of empty straight and corrugated cylindrical tubes are described in Table 5. According to Table 5, peak force of corrugated tubes varies between 74.07 kN for Cr3n20 and 113.74 kN for Cr18n8, while it is set 239.85 kN for the straight tube. Another imperative advantage of corrugation is the fact that these models have increased crushing length. It is figured out that Cr5n16 presents the highest value of SEA of corrugated tubes while it has appropriate CFE. In comparison with the straight tube, this model improves 149.86% of CFE while

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their mean force are approximately the same, and SEA decreases 10.86%.



Figure 7. Force-displacement curves for empty straight and corrugated cylindrical tubes.

Table 5. A comparison between crashworthiness parame-
ters of empty straight and corrugated cylindrical tubes

		-		
Code	Peak	Mean	SEA	CEE
	force(kN)	force(kN)	(kj/kg)	CLE
Cr18n8	113.74	27.45	33.56	0.2413
Cr9n12	111.71	31.14	37.22	0.2787
Cr5n16	94.38	34.62	39.39	0.3668
Cr3n20	74.07	29.46	29.21	0.3978
Straight	239.85	35.21	44 19	0 1468
tube	237.05	93.20	11.17	0.1+00

Fig. 8 indicates the force-displacement curves for empty straight and corrugated conical tubes subjected to axial crushing. As demonstrated in this figure, the maximum deflection is about 65 mm for straight tube while it is more than 80 mm for corrugated ones in conical tubes. The crashworthiness parameters of empty straight and corrugated conical tubes are described in Table 6. According to Table 6, the mean force and CFE have been improved significantly in corrugated tubes in comparison with straight conical one. On the other hand, a comparison of SEA between corrugated tubes and straight one depicts that straight tube has the highest value of SEA. According to Table 6, SEA of corrugated tubes varies between 29.82 kJ/kg for CCr3n20 and 42.64 kJ/kg for CCr18n8, while it is set 45.91 kJ/kg for the straight conical tube. It is figured out that CCr18n8 presents the highest value of SEA of corrugated tubes. However CCr3n20 presents the highest value of CFE of corrugated tubes which is 88.06% higher in

comparison with straight cylindrical tube. Furthermore CCr5n16 improves CFE by about 70% in comparison with the straight tube without an intense reduction (about 12%) in the SEA.

empty straight and corrugated conical tubes					
Code	Peak	Mean	SEA	CEE	
	force(kN)	force(kN)	(kj/kg)	CFE	
CCr18n8	119.80	34.88	42.64	0.2912	
CCr9n12	89.41	33.60	40.17	0.3758	
CCr5n16	73.62	35.40	40.27	0.4808	
CCr3n20	56.31	30.07	29.82	0.5341	
Straight	129.70	26 57	45.01	0 2840	
tube	128.79	30.37	45.91	0.2840	





In order to have better comparison between conical and cylindrical corrugated tubes, SEA and CFE of the proposed tubes are compared in Fig. 9 and Fig. 10 respectively. As indicated in these figures, the conical and cylindrical tubes show the same trend by variation the number of corrugations. Moreover, in all the models, the conical corrugated tubes have better performance than cylindrical ones in case of SEA and CFE. For instance the SEA is improved by 27% in CCr18n8 in comparison with Cr18n8. Moreover the CFE is improved by 21-34% in conical tubes compared to cylindrical ones.



Figure 9. A comparison of SEA between cylindrical and conical corrugated tubes.



cal corrugated tubes.

4.3. Crashworthiness of foam-filled corrugated tubes

Based on the proposed parametric study, Cr5n16 presents higher performance in case of the SEA, and CFE between cylindrical tubes. Hence the proposed structure is considered to be filled with different aluminum foams in order to investigate the effect of foams on the crashworthiness of cylindrical corrugated composite tubes. Three different aluminum foams are applied in this paper. Fig. 11 indicates a comparison between force-displacement curves of empty and foam-filled cylindrical composite corrugated tube. As shown in Fig. 11, using foams improves energy absorption of structures. However, SEA has been decreased significantly due to the considerable increase in mass.





According to Fig. 12 that presents the effect of foam density on crashworthiness parameters of cylindrical corrugated tubes, using foam with a higher density increase the mean force and SEA impressively. However, the peak force has been increased undesirably during the deflection due to the fact that in higher densities, densification region accrues in fewer strains. As indicated in Fig. 11, structures with higher-density foam represent unsuitable crushing behavior, and it is not appropriate for crashworthiness purposes. On the other hand, the tube which is filled with low-density foam provides a smooth force-displacement curve, and therefore it has the highest value of CFE. However, the SEA of this model is unacceptable in comparison with the empty tube.



Figure 12. Effect of foam density on crashworthiness parameters of Cr5n16.

Similar to cylindrical tubes, CCr5n16 is considered to be filled with different aluminum foams in order to

investigate the effect of foams on the crashworthiness of conical corrugated composite tubes. Fig. 13 indicates a comparison between force-displacement curves of empty and foam-filled conical composite corrugated tube. It is clear in this figure that by increasing the foam density, the absorbed energy is rose significantly.



Figure 13. A comparison between force-displacement curves of empty and foam-filled conical composite corrugated tubes.

Fig. 14 indicates the effect of foam density on crashworthiness parameters of conical corrugated tubes. It is demonstrated from Fig. 14, the peak force is increased by increasing the foam density. However, the highest value of CFE and mean force is achieved in the model which is filled with medium

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density foam. Hence medium density foam has better performance in case of both SEA and CFE.



ters of CCr5n16.

5. conclusion

This study was carried out to investigate the crushing behavior of empty and foam-filled cylindrical and conical corrugated composite tubes under axial crushing. Different corrugated tubes have been compared with straight ones. Foam-filled structures with different foam densities are studied regarding specific energy absorption and CFE. Based on the obtained results the following points can be concluded.

• The numerical model in LS-DYNA software shows good agreement with experiment however some modes of failure of composites including delamination could not predict with this model.

• Creating corrugated surfaces on tubes, have improved the crush force efficiency significantly in all models in comparison with straight tubes.

• The corrugated cylindrical model (Cr5n16) was achieved that improves CFE by about 150% in comparison with the straight tube without an intense reduction (about 11%) in the SEA.

• The corrugated conical model (CCr5n16) was achieved that improves CFE by about 70% in comparison with the straight tube without an intense reduction (about 12%) in the SEA.

• Overall, the absorbed energy is increased by using foams in both conical and cylindrical tubes. In cylindrical tubes, by increasing the

foam density CFE is considerably decresed due to to the fact that in higher densities, densification region accrues in fewer strains. Moreover, medium density foam has better performance in case of both SEA and CFE for conical corrugated tubes.

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